Section 3.3 **Derivatives of trigonometric functions**

Theorem. $\boxed{\lim_{\theta \to 0} \sin \theta = 0.}$

Theorem. $\boxed{\lim_{\theta \to 0} \cos \theta = 1.}$

Theorem. $\boxed{\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1}$

Corollary. $\boxed{\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0.}$

Derivatives

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx} \cot x = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$y = \sin x$

$y' - ?$

$\boxed{\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}$

$\boxed{\lim_{\heartsuit \to 0}\frac{\sin\heartsuit}{\heartsuit} = 1}$

$$\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0}\frac{2\cos\frac{x+h+x}{2}\sin\frac{x+h-x}{2}}{h}$$

$$= \lim_{h\to 0}\frac{2\cos\frac{2x+h}{2}\sin\frac{h}{2}}{h} = \lim_{h\to 0}\frac{2\cos\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= \cos x \cdot \lim_{h\to 0}\frac{2\sin\frac{h}{2}}{h} = \cos x \cdot \lim_{h\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}} = \cos x$$

$$\boxed{(\sin x)' = \cos x}$$

$$\boxed{(\cos x)' = -\sin x}$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)'\cos x - (\cos x)'\sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Example 1. Find $\frac{dy}{dx}$

1. $y = \cos x - 2\tan x$

$y' = (\cos x)' - 2(\tan x)'$

$= -\sin x - 2\sec^2 x$

2. $y = 2x(\sqrt{x} - \cot x)$ $= 2x\sqrt{x} - 2x\cot x$

$= 2x^{3/2} - 2x\cot x$

$y' = 2 \cdot \frac{3}{2} x^{3/2 - 1} - [2(x)'\cot x + 2x(\cot x)']$

$= \boxed{3x^{1/2} - 2\cot x + 2x\csc^2 x}$

3. $y = x\csc x$

$y' = (x)'\csc x + x(\csc x)'$

$= \csc x + x(-\csc x\cot x)$

$= \boxed{\csc x - x\csc x\cot x}$

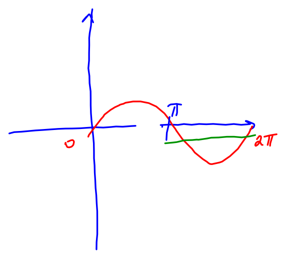
Example 2. Find the points on the curve $y = \frac{\cos x}{2 + \sin x}$ at which the tangent is horizontal, $0 \le x \le 2\pi$

horizontal tangent = slope is zero

$$y' = \frac{(\cos x)'(2+\sin x) - \cos x\,(2+\sin x)'}{(2+\sin x)^2}$$

$$= \frac{-\sin x(2+\sin x) - \cos x(\cos x)}{(2+\sin x)^2} = \frac{-2\sin x \underbrace{- \sin^2 x - \cos^2 x}_{-1}}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2} = 0$$



$$-2\sin x - 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$x_1 = \pi + \frac{\pi}{6} = \boxed{\frac{7\pi}{6}}$$

$$x_2 = 2\pi - \frac{\pi}{6} = \boxed{\frac{11\pi}{6}}$$