## Section 3.3 Derivatives of trigonometric functions

$$
\text { Theorem. } \lim _{t \rightarrow 0} \sin \theta=0 \text {. }
$$

$$
\text { Theorem. } \lim _{\theta \rightarrow 0} \cos \theta=1
$$

Theorem. $\lim _{\theta \rightarrow 0} \frac{\sin \theta}{\theta}=1$

Corollary. $\lim _{\theta \rightarrow 0} \frac{\cos \theta-1}{\theta}=0$.

## Derivatives

$$
\begin{gathered}
\frac{d}{d x} \sin x=\cos x \\
\frac{d}{d x} \cos x=-\sin x \\
\frac{d}{d x} \tan x=\frac{1}{\cos ^{2} x}=\sec ^{2} x \\
\frac{d}{d x} \cot x=-\frac{1}{\sin ^{2} x}=-\csc ^{2} x \\
\frac{d}{d x} \csc x=-\csc x \cot x \\
\frac{d}{d x} \sec x=\sec x \tan x
\end{gathered}
$$

$$
\begin{aligned}
& y=\sin x \\
& \begin{array}{r}
y^{\prime}-\text { ? } \\
\sin
\end{array} \\
& \lim _{h \rightarrow 0} \frac{y-? ~}{\sin (x+h)-\sin x}<h \quad \lim _{h \rightarrow 0} \frac{2 \cos \frac{x+h+x}{2} \sin \frac{x+h-x}{2}}{h} \\
& \lim _{\Delta \rightarrow 0} \frac{\sin s}{\rho}=1 \\
& =\lim _{h \rightarrow 0} \frac{2 \cos \frac{2 x+h}{2} \sin \frac{h}{2}}{h}=\lim _{h \rightarrow 0} \frac{2 \cos \left(x+\frac{h}{h}\right) \sin \frac{h}{2}}{h} \\
& =\cos x \cdot \lim _{h \rightarrow 0} \frac{\sqrt{2 \sin } \frac{h}{2}}{h}=\cos x \cdot \lim _{h \rightarrow 0} \frac{\sin \frac{h^{\prime}}{2}}{\frac{h}{2}}=\cos x \\
& (\sin x)^{\prime}=\cos x \\
& (\cos x)^{\prime}=-\sin x \\
& (\tan x)^{\prime}=\left(\frac{\sin x}{\cos x}\right)^{\prime}=\frac{(\sin x)^{\prime} \cos x-(\cos x)^{\prime} \sin x}{\cos ^{2} x}=\frac{\cos ^{2} x+\sin ^{2} x}{\cos ^{2} x}=\frac{1}{\cos ^{2} x}=\sec ^{2} x
\end{aligned}
$$

Example 1. Find $\frac{d y}{d x}$

1. $y=\cos x-2 \tan x$

$$
\begin{aligned}
y= & \cos x-2 \tan x \\
y^{\prime} & =(\cos x)^{\prime}-2(\tan x)^{\prime} \\
& =-\sin x-2 \sec ^{2} x
\end{aligned}
$$

2. $\begin{aligned} y=2 x(\sqrt{x}-\cot x) & =2 x \sqrt{x}-2 x \cot x \\ & =2 x^{3 / 2}-2 x \cot x\end{aligned}$

$$
\begin{aligned}
y^{\prime} & =2 \frac{3}{2} x^{3 / 2-1}-\left[2(x)^{\prime} \cot x+2 x(\cot x)^{\prime}\right] \\
& =3 x^{1 / 2}-2 \cot x+2 x \csc ^{2} x
\end{aligned}
$$

3. $y=x \csc x$

$$
\begin{aligned}
y^{\prime} & =(x)^{\prime} \csc x+x(\operatorname{crc} x)^{\prime} \\
& =\csc x+x(-\csc x \cot x) \\
& =\csc x-x \operatorname{crc} x \cot x
\end{aligned}
$$

Example 2. Find the points on the curve $y=\frac{\cos x}{2+\sin x}$ at which the tangent is horizontal. $0 \leq x \leq 2 \pi$
horizontal tangent $=$ slope, is zero

$$
\begin{gathered}
y^{\prime}=\frac{(\cos x)^{\prime}(2+\sin x)-\cos x(2+\sin x)^{\prime}}{(2+\sin x)^{2}}=\frac{-\sin x(2+\sin x)-\cos x(\cos x)}{(2+\sin x)^{2}}=\frac{-2 \sin x-\sin ^{2} x-\cos ^{2} x}{(2+\sin x)^{2}} \\
=\frac{-2 \sin x-1}{(2+\sin x)^{2}}=0
\end{gathered}
$$

$$
-2 \sin x-1=0
$$

$$
\sin x=-\frac{1}{2}
$$



$$
x_{1}=\pi+\frac{\pi}{6}=\frac{7 \pi}{6}
$$

$$
x_{2}=2 \pi-\frac{\pi}{6}=\frac{11 \pi}{6}
$$

