Section 3.4 **The Chain Rule**

> If the derivatives $g'(x)$ and $f'(g(x))$ both exist, and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then $F'(x)$ exists an is given by the product
>
> $$\boxed{[f(g(x))]' = F'(x) = f'(g(x))g'(x)}$$

Example 1. If $f(x) = x^2 + 4x - 3$ and $g(x) = \sin x$, write equations for $f(g(x))$ and $g(f(x))$.

Example 2. Suppose that $F(x) = f(g(x))$, where $g(2) = 5$, $g'(2) = 4$, $f(2) = 3$, $f'(2) = -2$, and $f'(5) = 11$. Find $F'(2)$.

> If n is any real number and $u = g(x)$ is differentiable, then
>
> $$\boxed{\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)}$$

> If n is any real number and $u = g(x)$ is differentiable, then
>
> $$\boxed{\frac{d}{dx}e^{g(x)} = g'(x)e^{g(x)}}$$

Example 3. Find the derivative of each function

1. $y = \sec(2x)$

2. $y = \sin(x^2)$

3. $y = (1 + \cos^2 x)^6$

4. $y = (1 + \sqrt{x^2 + 2})^3$

5. $y = \sqrt[4]{\dfrac{t^3 + 1}{t^3 - 1}}$

6. $y = \sin^2(\cos 4x)$

7. $y = e^{x \sin 2x}$

Example 4. Find the equation of the tangent line to the curve $y = \dfrac{8}{\sqrt{4+3x}}$ at the point (4,2).