Section 3.5 **Implicit differentiation.**

Some functions are defined implicitly by a relation between x and y, where x is the independent variable and y depends on x. In order to find the derivative of y with respect to x, we can use the method of **implicit differentiation**. This consists of differentiating both sides of the relation with respect to x and then solving the resulting equation for y'.

Example 1. Find dy/dx by implicit differentiation.

1. $x^2 - xy + y^3 = 8$

2. $xe^y = x - y$

3. $\tan(x-y) = \dfrac{y}{1+x^2}$

Example 2. Let y be the independent variable and x be the dependent variable. Use implicit differentiation to find dx/dy if

$$(x^2+y^2)^2 = 4x^2y$$

Derivatives of the inverse trigonometric functions.

Let us find $(\arcsin x)'$:

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$$

Example 3. Find the derivative.

1. $f(x) = \arccos(x^2)$

2. $f(x) = \arctan\sqrt{\dfrac{1-x}{1+x}}$