Section 3.6 **Derivatives of logarithmic functions**

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}} \qquad \boxed{\frac{d}{dx}\log_a x = \frac{1}{x\ln a}}$$

$$\boxed{\frac{d}{dx}\ln g(x) = \frac{g'(x)}{g(x)}}$$

Example 1. Differentiate each function

1. $f(x) = \ln(\sin^2 x)$

2. $\dfrac{d}{dx}\log_3(\tan x^2)$

3. $f(x) = \ln|x|$

4. $f(x) = \sqrt{\dfrac{9-x^2}{9+x^2}}$

Example 2. Find y' and y'' if

$$y = \sqrt{x}\,\ln x.$$

$$\boxed{a^x = e^{x\ln a}}$$

$$\boxed{(a^x)' = a^x \ln a}, \qquad a > 0, a \neq 1.$$

Example 3. Find $\dfrac{d}{dx}\left(\sqrt{2-3^x} + \pi^{-x} + x^e\right)$

Logarithmic differentiation

Steps in logarithmic differentiation

1. Take the logarithm of both sides of an equation.
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y'.

Example 4. Differentiate each function

1. $y = x^x$

2. $y = x^{\sin x}$

3. $y = \cos(x^{\sqrt{x}})$

4. $y = \dfrac{(x+1)^4 \sqrt[5]{x^2+1}}{(x^3-1)^{151}(1+3x^2)^{2018}}$

Example 5. Find y' if $x^y = y^x$.