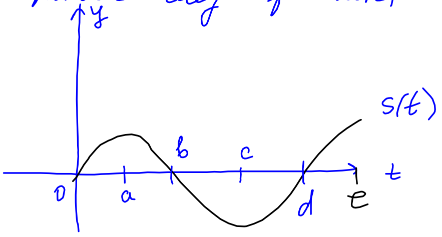
3.7. Rates of Change in the natural and social sciences.

a particle that moves with the position function $s(t)$, then its velocity is $\boxed{v(t)=s'(t)}$

acceleration $\boxed{a(t)=v'(t)=s''(t)}$

E.1. Given the graph of the position function $s(t)$ for a particle that moves horizontally forward



(a) When is the particle moving forward

$v(t)>0$ or $s(t)$ is increasing

$\boxed{(0,a)\cup(c,e)}$

(b) when is the particle moving backward?

$v(t)<0$ or $s(t)$ is decreasing

$\boxed{(a,c)}$

(c) when is the particle at rest?

slope $=0$ (horizontal tangents)

$\boxed{t=a \ \& \ t=c}$

E.2. an object is thrown vertically upward with the velocity of 40 ft/s. Its height after t sec is given by the equation

$$h(t)= 40t-16t^2+24.$$

(a) What is the max height reached by the object?

when $v(t)=0$

$$v(t)= h'(t) = (40t-16t^2+24)'$$
$$= 40-32t=0$$
$$t = \frac{40}{32} = \frac{5}{4} \text{ sec}$$

$$h\left(\frac{5}{4}\right) = 40\cdot\frac{5}{4} - 16\cdot\left(\frac{5}{4}\right)^2 + 24$$
$$= 50-25+24 = \boxed{49 \ (ft)}$$

(b) What is the object's velocity when it is at a height of 40 ft on its way down?

Find t such that $h(t)=40$

$$\frac{40t-16t^2+24}{8} = \frac{40}{8} \Rightarrow 5t-2t^2+3=5 \ \Big| \ 5t-2t^2-2=0$$
$$2t^2-5t+2=0$$
$$t_1 = \frac{5+\sqrt{25-16}}{4} = \frac{5+3}{4} = 2$$
$$t_2 = \frac{5-3}{4} = \frac{1}{2}$$

$$\underline{\boxed{t=2}}$$

$$v(2) = 40-32(2) = \boxed{-24 \ ft/s}$$

(c) With what velocity it will hit the ground?

hits the ground when $h(t)=0$

$$\frac{40t-16t^2+24}{8} = \frac{0}{8}$$
$$5t-2t^2+3=0 \quad \text{or} \quad 2t^2-5t-3=0$$
$$t_1 = \frac{5+\sqrt{25+24}}{4} = \frac{5+7}{4} = \boxed{3=t}$$
$$t_2 = \frac{5-7}{4} = \frac{-1}{2}$$

$$v(3) = 40-32(3) = \boxed{-56 \ ft/s}$$

E.3. a particle moves according to the position function $s(t) = 2t^3 - 12t^2 + 18t + 3$, for $t \geq 0$, t is in sec, s is in feet.

(a) when is the particle at rest?

when $v(t) = 0$

$$s'(t) = v(t) = \frac{6t^2 - 24t + 18}{6} = \frac{0}{6}$$

$$t^2 - 4t + 3 = 0$$

$$(t-3)(t-1) = 0 \Rightarrow \boxed{t = 3, t = 1}$$

(b) when is the particle moving in the positive direction? negative direction?

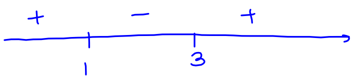
positive direction when $v(t) > 0$

negative $v(t) < 0$

$6t^2 - 24t + 18 > 0$

$t^2 - 4t + 3 > 0$

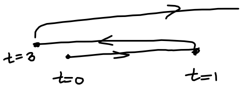
$(t-1)(t-3) > 0$



$v(0) > 0$

$v(t) > 0$ on $(0,1) \cup (3, \infty)$

$v(t) < 0$ on $(1,3)$



(c) Find the total distance travelled in the first 5 sec.

from $t=0$ to $t=1$ $\quad d_1 = s(1) - s(0) = \underbrace{2(1)^3 - 12(1)^2 + 18(1) + 3}_{s(1)} - \underbrace{3}_{s(0)}$

$= 8$

from $t=1$ to $t=3$ $\quad d_2 = s(3) - s(1) = 2(3)^3 - 12(3)^2 + 18(3) + 3 - 11 = -8$

from $t=3$ to $t=5$ $\quad d_3 = s(5) - s(3) = 2(5)^3 - 12(5)^2 + 18(5) + 5 - 3 = 42$

total distance $d = d_1 + |d_2| + d_3 = 8 + 8 + 42 = \boxed{58 \text{ ft}}$

E. 4. a spherical baloon is being blown up. Find the rate at which the volume of the sphere changes with respect to the radius, when the radius of the baloon is 2in.

$$V = \frac{4}{3}\pi r^3$$

rate of change is $\frac{dV}{dr} = \frac{4}{3}\pi(3r^2)$

$$= 4\pi r^2$$

$$\frac{dV}{dr}(2) = 4\pi \cdot 2^2 = \boxed{16\pi}$$