Section 3.8 **Exponential growth and decay**

If $y(t)$ is the value of a quantity y at time t and if the rate of change of y with respect to t is proportional to $y(t)$ at any time, then

$$\frac{dy}{dt} = ky$$

where k is a constant. This equation is called the **law of natural growth** if $k > 0$ or the the **law of natural decay** if $k < 0$.
The only solution to this equation is

$$\boxed{y(t) = y(0)\mathrm{e}^{kt}}$$

Example 1. A bacteria culture starts with 500 bacteria and after 3 hours there are 8000 bacteria.

1. Find an expression for the number of bacteria after t hours.

2. Find the number of bacteria after 4 hours.

3. When will the population reach 30,000?

Example 2. Polonium-214 has a half-life of 1.4×10^{-4} s.

1. If a sample has a mass of 50 mg, find a formula for the mass that remains after t seconds.

2. Find the mass that remains after a hundredth of a second.

3. How long would it take for the mass to decay to 40 mg?

Newtons Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings.

$$\frac{dy}{dt} = k(y - M)$$

where y is the temperature of the object and M the temperature of the surroundings. The solution of this equation is a function of the form:

$$\boxed{y(t) = (y_0 - M)e^{kt} + M}$$

where y_0 is the initial temperature of the object.

Example 3. A roast turkey is taken from the oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F.

1. If the temperature of turkey is 150°F after half an hour, what is the temperature after 45 min?

2. When will the turkey have cooled to 100°F ?