Section 4.1 **Maximum and minimum values**

Definition. Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum value** of f on D, if $f(c) \geq f(x)$ for all x in D.
- **absolute minimum value** of f on D, if $f(c) \leq f(x)$ for all x in D.

Definition. A number $f(c)$ is a

- **local maximum value**, if $f(c) \geq f(x)$ when x is near c.
- **local minimum value**, if $f(c) \leq f(x)$ when x is near c.

Example 1. Sketch the graph of the function f that is continuous on $[0, 3]$ and has the absolute maximum at 0, absolute minimum at 3, local minimum at 1, local maximum at 2.

The extreme value theorem. If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Fermat's theorem. If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$

Definition. A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Example 2. Find the critical numbers of the function.

(a.) $f(x) = 4x^3 - 9x^2 - 12x + 3$

(b.) $f(x) = \dfrac{x}{x^2 + 1}$

If f has a local extremum at c, then c is a critical number of f.

The closed interval method. To find the **absolute** maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b)
2. Find $f(a)$ and $f(b)$
3. The largest number of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Example 3. Find the absolute maximum and absolute minimum values of f on the given interval.

(a.) $f(x) = x^2 + \dfrac{2}{x}$, $[1/2, 2]$

(b.) $f(x) = \cos x + \sin x$, $[0, \pi/3]$

(c.) $f(x) = x\mathrm{e}^{-x}$, $[0, 2]$