Section 4.2 **The mean value theorem.**

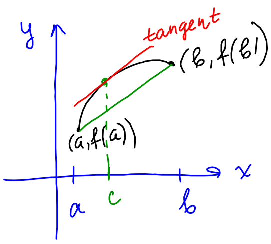
The mean value theorem. Let f be a function that satisfies the following hypotheses:

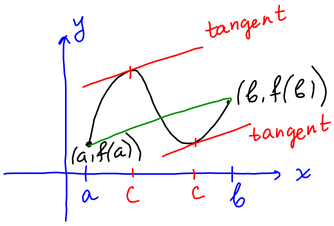
- f is continuous on the closed interval $[a, b]$.
- f is differentiable on an open interval (a, b).

Then there is a number c such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

Meaning: There is at least one value c in (a, b), where the tangent line at c is parallel to the secant line between $(a, f(a))$ and $(b, f(b))$.





Example 1. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers that satisfy the conclution of the Mean Value Thorem.

1. $f(x) = x^3 - 3x + 2$, $[-2, 2]$. continuous on $(-\infty, \infty)$
differentiable on $(-\infty, \infty)$

Find a number c such that

$f'(c) = \dfrac{f(2) - f(-2)}{2-(-2)} = \dfrac{4-0}{4} = 1$

$f'(x) = 3x^2 - 3$

$f'(c) = 3c^2 - 3 = 1$

$3c^2 = 4$

$\boxed{c = \pm \frac{2}{\sqrt{3}}}$

2. $f(x) = \ln x$, $[1, 4]$. continuous on $(0, \infty)$

$f'(x) = \frac{1}{x}$ exists for all $x \neq 0$.

$f(x) = \ln x$, differentiable on $(0, \infty)$

$f'(c) = \frac{1}{c} = \dfrac{f(4) - f(1)}{4-1}$

$\frac{1}{c} = \frac{\ln 4}{3} \Rightarrow \boxed{c = \frac{3}{\ln 4}}$