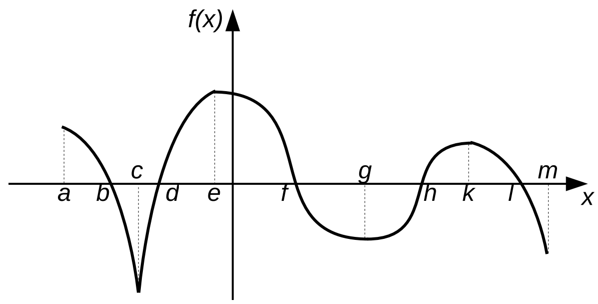
Section 4.3 **How derivatives affect the shape of the graph.**

- If $f'(x) > 0$ on an interval, then f is increasing on that interval
- If $f'(x) < 0$ on an interval, then f is decreasing on that interval
- f has a **local maximum** at the point, where its derivative changes from positive to negative.
- f has a **local minimum** at the point, where its derivative changes from negative to positive.

Example 1. Given the graph of the function f.



(a.) What are the x-coordinate(s) of the points where $f'(x)$ does not exist?

(b.) Identify intervals on which $f'(x) > 0$.

$f'(x) < 0$.

(c.) Identify the x coordinates of the points where $f(x)$ has a local maximum.

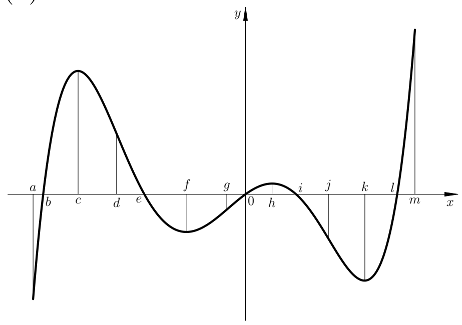
A local minimum.

What does f'' say about f?

- If $f''(x) > 0$ on an interval, then f is **concave upward (CU)** on that interval
- If $f''(x) < 0$ on an interval, then f is **concave downward (CD)** on that interval

Definition. A point where curve changes its direction of concavity is called an **inflection point**

Example 2. Given the graph of $f'(x)$.



(a.) Identify intervals on which f is increasing.

Is decreasing.

(b.) Identify the x coordinates of the points where f has a local maximum.

A local minimum.

(c.) Identify intervals on which f is concave upward.

Concave downward.

(d.) Find the x-coordinates of inflection points.

Example 3. Sketch the graph of a function that satisfies all of the given conditions.

1. $f'(5) = 0$
2. $f'(x) < 0$ when $x < 5$
3. $f'(x) > 0$ when $x > 5$,
4. $f''(2) = f''(8) = 0$
5. $f''(x) < 0$ when $x < 2$ or $x > 8$
6. $f''(x) > 0$ when $2 < x < 8$
7. $\lim_{x \to \infty} f(x) = 3$
8. $\lim_{x \to -\infty} f(x) = -3$

The second derivative test. Suppose f'' is continuous near c.

1. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local min at c.
2. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local max at c.

bf Example 4. For the given functions, find the following:

1. Domain
2. Asymptotes
3. Intercepts
4. Intervals of increase/decrease
5. Local Extrema
6. Intervals of Concavity
7. Inflection Points
8. Sketch a graph

$f(x) = x^4 - 6x^2$

$$f(x) = \frac{x}{(x-1)^2}$$

$f(x) = e^{-\frac{1}{x+1}}$