Section 4.9 **Antiderivatives**

Definition. Function $F(x)$ is called an
bf antiderivative of $f(x)$ on an interval I if

$$F'(x) = f(x)$$

for all $x \in I$.

Theorem 1. If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is a constant.

Table of antidifferentiation formulas

Function	Antiderivative
$af(x)$, a is a constant	$aF(x) + C$
$f(x) + g(x)$	$F(x) + G(x) + C$
a, a is a constant	$ax + C$
x	$\frac{x^2}{2} + C$
x^n, $n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln \lvert x \rvert + C$
e^x	$e^x + C$
a^x	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} + C$

Example 1. Find the most general antiderivative of the function.

(a.) $f(x) = x^3 - 4x^2 + 17$

(b.) $f(t) = \sin t - \sqrt{t}$

(c.) $f(x) = (1 + x^2)\sqrt[3]{x^2}$

(d.) $f(x) = \dfrac{x^2 + x + 1}{x}$

(e.) $f(x) = x^e + \dfrac{5}{1 + x^2} - \dfrac{1}{\sqrt{1 - x^2}}$

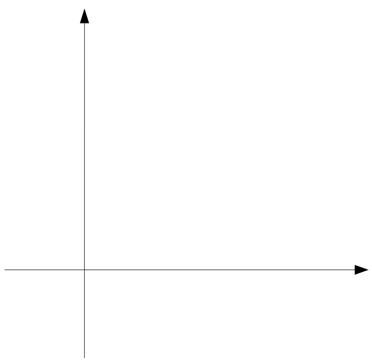
Example 2. Find $f(x)$ if

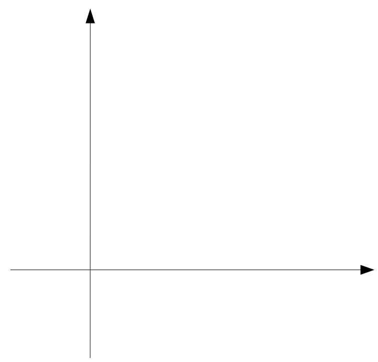
(a.) $f'(x) = 3\sqrt{x} - \dfrac{1}{\sqrt{x}}$, $f(1) = 2$

(b.) $f''(x) = x,\ f(0) = -3,\ f'(0) = 2$

The geometry of antiderivatives

Example 3. Given the graph of a function $f(x)$. Make a rough sketch of of an antiderivative of F, given that $F(0) = 0$.

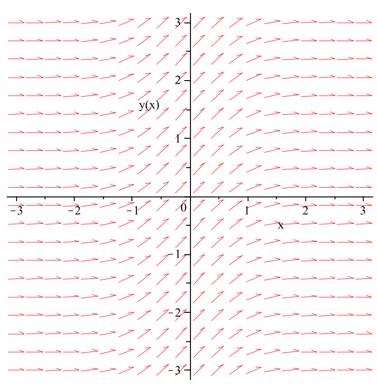


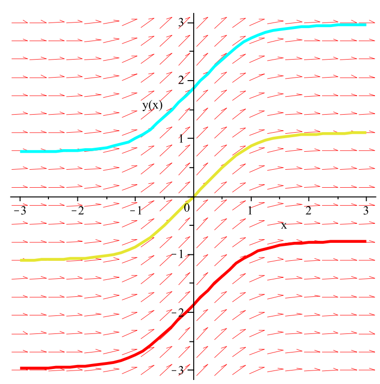


Example 4. If

$$f(x) = \frac{1}{x^4 + 1},$$

sketch the graph of those antiderivatives F that satisfy the initial conditions $F(-1) = 1$, $F(0) = 0$, $F(1) = -1$.





Rectilinear motion

If the object has a position function $s = s(t)$, then

$v(t) = s'(t)$ (the position function is an antiderivative for the velocity function),

$a(t) = v'(t)$ (the velocity function is an antiderivative to the acceleration function)

Example 5. A particle is moving with the acceleration $a(t) = 3t + 8$, $s(0) = 1$, $v(0) = -2$. Find the position of the particle.

Antiderivatives of vector functions

Definition. A vector function $\mathbf{R}(t) =< X(t), Y(t) >$ is called **an antiderivative** of $\mathbf{r}(t) =< x(t), y(t) >$ on an interval I if $\mathbf{R}'(t) = \mathbf{r}(t)$ that is, $X'(t) = x(t)$ and $Y'(t) = y(t)$.

Theorem 2. If $\mathbf{R}$ is an antiderivative of $\mathbf{r}$ on an interval I, then the most general antiderivative of $\mathbf{r}$ on I is

$$\mathbf{R} + \mathbf{C}$$

where $\mathbf{C}$ is an arbitrary constant vector.

Example 6. Find the vector-function that describe the position of particle that has an acceleration $\mathbf{a}(t) = \cos t\mathbf{i} + e^t\mathbf{j}$ and $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$, $\mathbf{r}(0) = \mathbf{0}$.