Section 5.3 **The fundamental theorem of calculus.**

Theorem. Suppose f is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$.
2. $\boxed{\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)}$, where F is an antiderivative of f.

Example 1. Find the derivative of the function.

1. $g(x) = \int_\pi^x \frac{1}{1+t^4}dt$

2. $f(x) = \int_x^4 (2+\sqrt{t})^8 dt$

3. $y = \int_{\tan x}^{17} \sin(t^4)dt$

Example 2. Evaluate the integral.

1. $\displaystyle\int_2^6 \frac{1+\sqrt{y}}{y^2}dy$

2. $\displaystyle\int_0^4 (4-t)\sqrt{t}\,dt$

3. $\displaystyle\int_0^3 (2\sin x - e^x), dx$

4. $\displaystyle\int_0^4 2^s, ds$

5. $\displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}, dx$

6. $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}, dx$

7. $\int_0^2 f(x)dx,$ where $f(x) = \begin{cases} x^4 & 0 \le x < 1 \\ x^5 & 1 \le x \le 2 \end{cases}$

Example 3. Find the area of the region enclosed by the parabola $y = 2x - x^2$ and the line $y = 0$.