

## Section 6.5 The Substitution Rule

**The substitution rule for indefinite integrals** If  $u = g(x)$  is a differentiable function whose range is an interval  $I$  and  $f$  is continuous on  $I$ , then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

**Example 1.** Evaluate each integral:

1.  $\int x^2 e^{x^3} dx$

2.  $\int \frac{x + \arcsin x}{\sqrt{1-x^2}} dx$

3.  $\int \sin(5x + 4) dx$

4.  $\int \frac{dx}{\sqrt{3x+1}}$

$$5. \int \frac{2x^2 + 4x}{x^3 + 3x^2 - 4} dx$$

$$6. \int x^2 \sqrt{2+x} dx$$

$$7. \int x^3 \sqrt{x^2 + 1} dx$$

**The substitution rule for definite integrals** If  $g'(x)$  is continuous on  $[a, b]$  and  $f$  is continuous on the range of  $g$ , then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

**Example 2.** Evaluate the integral:

$$1. \int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

$$2. \int_0^1 \frac{x dx}{\sqrt{1+x^4}}$$

$$3. \int_0^1 \frac{e^z + 1}{e^z + z} dz$$

**Integrals of symmetric functions** Suppose  $f$  is continuous on  $[-a, a]$ .

(a) If  $f$  is **even**, that is  $f(-x) = f(x)$ , then  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

(b) If  $f$  is **odd**, that is  $f(-x) = -f(x)$ , then  $\int_{-a}^a f(x) dx = 0$

**Example 4.** Evaluate the integral  $\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6} dx$ .