

Slopes and tangents to parametric curves

Suppose that the curve C is given by parametric equations $x = x(t)$, $y = y(t)$, then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}$$

Example 1. Find an equation of the tangent to the curve $x(t) = t \sin t$, $y(t) = t \cos t$ at the point corresponding to $t = \pi$.

Example 2. Find the points on the curve $x = t(t^3 - 3)$, $y = 3(t^3 - 3)$, where the tangent is vertical or horizontal.

Example 3. At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with the equations $x = -7t$, $y = 12t - 5$?