Sample Problems for the Final

- 1. Given vectors $\mathbf{a} = \mathbf{i} 2\mathbf{j}$, $\mathbf{b} = \langle -2, 3 \rangle$. Find
 - (a) a unit vector **u** that has the same direction as $2\mathbf{b} + \mathbf{a}$.
 - (b) angle between **a** and **b**
 - (c) comp_b**a**, proj_b**a**.
- 2. Find the work done by a force of 20 lb acting in the direction N50°W in moving an object 4 ft due west.
- 3. Find the distance from the point (-2,3) to the line 3x 4y + 5 = 0.
- 4. Find vector and parametric equations for the line passing through the points A(1, -3) and B(2, 1).
- 5. Find all points of discontinuity for the function

$$f(x) = \begin{cases} x^2 + 1 & , & \text{if } x < 2, \\ x + 2 & , & \text{if } x \ge 2. \end{cases}$$

6. Find the vertical and horizontal asymptotes of the curve $y = \frac{x^2 + 4}{3x^2 - 3}$.

- 7. Find $\frac{dy}{dx}$ for each function
 - (a) $y = (\sin x)^x$. $\sqrt[5]{2x-1}(x^2-4)^2$

(b)
$$y = \frac{\sqrt[3]{2x - 1(x^2 - 4)^2}}{\sqrt[3]{1 + 3x}}$$

- (b) $y = \frac{\sqrt[3]{1+3x}}{\sqrt[3]{1+3x}}$ (c) $y(t) = \sin^{-1} t, x(t) = \cos^{-1}(t^2).$
- (d) $2x^2 + 2xy + y^2 = x$.
- 8. Find the equation of the tangent line to the curve $y = x\sqrt{5-x}$ at the point (1,2).
- 9. A particle moves on a vertical line so that its coordinate at time t is $y = t^3 12t + 3$, $t \ge 0$.
 - (a) Find the velocity and acceleration functions.
 - (b) When is the particle moving upward?
 - (c) Find the distance that particle travels in the time interval $0 \le t \le 3$
- 10. The vector function $\mathbf{r}(t) = \langle t, 25t 5t^2 \rangle$ represents the position of a particle at time t. Find the velocity, speed, and acceleration at t = 1.
- 11. Find y'' if $y = e^{-5x} \cos 3x$

12. Find
$$\frac{d^{50}}{dx^{50}}\cos 2x$$

- 13. Use differentials to estimate $(1.09)^{10}$.
- 14. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
- 15. Evaluate each limit:

(a)
$$\lim_{x \to 0} \frac{\sin x + \sin 2x}{\sin 3x}$$

- (b) $\lim_{x \to 0} (\cot x \csc x)$
- (c) $\lim_{x \to 0} x^{\sin x}$
- 16. A cup of coffee has a temperature of 200°F and is in a room that has a temperature of 70°F. After 10 min the temperature of the coffee is 150°F.
 - (a) What is the temperature of the coffee after 15 min?
 - (b) When will the coffee have cooled to 100° F?
- 17. Find the absolute maximum and absolute minimum values of $f(x) = x^3 2x^2 + x$ on [-1,1].
- 18. For the function $y = x^2 e^x$ find
 - (a) All asymptotes.
 - (b) Intervals on which the function is increasing/decreasing.
 - (c) All local minima/local maxima.
 - (d) Intervals on which the function is CU/CD.
 - (e) Inflection points.
- 19. The top and the bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of the printed material on the poster is fixed at 384 cm², find the dimensions of the poster with the smallest total area.
- 20. Find the area under the curve $y = x^2 + 3x 2$ from 1 to 4. Use equal subintervals and take x_i^* to be the right end-point of the *i*-th interval
- 21. Express the limit $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^n \frac{1}{1+(i/n)^2}$ as a definite integral. Do not evaluate it.

22. Find the derivative of the function
$$f(x) = \int_{0}^{\sqrt{x}} \frac{t^2}{t^2 + 1} dt$$

23. Find the integral:

(a)
$$\int_{1}^{2} \left(x + \frac{1}{x}\right)^{2} dx$$

(b)
$$\int_{1}^{2} \frac{x^{2} + 1}{\sqrt{x}} dx$$

(c)
$$\int_{0}^{\pi/2} (\cos t + 2\sin t) dt$$

(d)
$$\int \sqrt[3]{1 - x} dx$$

(e)
$$\int \frac{(1 + \sqrt{x})^{9}}{\sqrt{x}} dx$$

(f)
$$\int \frac{e^{x} + 1}{e^{x}} dx$$

(g)
$$\int_{0}^{4} \frac{x}{\sqrt{1+2x}} dx$$
 (HINT: do the substitution $u = 1+2x$)
(h)
$$\int_{1}^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^{2}}} dx$$

- 24. Find the area under the curve $y = \sqrt{x}$ above the x-axis between 0 and 4.
- 25. A particle moves in a straight line and has acceleration given by $a(t) = t^2 t$. Its initial velocity is v(0) = 2 cm/s and its initial displacement is s(0) = 1 cm. Find the position function s(t).
- 26. Find the vector function $\mathbf{r}(t)$ that gives the position of a particle at time t having the acceleration $\mathbf{a}(t) = 2t\mathbf{i} + \mathbf{j}$, initial velocity $\mathbf{v}(0) = \mathbf{i} \mathbf{j}$, and initial position (1,0).