Vector functions.

Definition. The curve of a type $x = x(t)$, $y = y(t)$ is called a **parametric curve** and the variable t is called a **parameter**.

Definition. Vector

$$\mathbf{r}(t) =< x(t), y(t) >= x(t)\mathbf{i} + y(t)\mathbf{j}$$

is called the **position vector** for the point with coordinates $(x(t), y(t))$.

A function such as $\mathbf{r}(t)$, whose range is a set of vectors, is called a **vector function** of t.

Example 1.

1. Sketch the curve represented by the parametric equation $x(t) = \dfrac{1-t}{1+t}$, $y = t^2$.

2. Eliminate the parameter to find the Cartesian equation of the curve.

Example 2. An object is moving in the xy-plane and its position after t seconds is $\mathbf{r}(t) = < t-3, t^2-2t >$.

1. Find the position of the object at time $t = 5$.

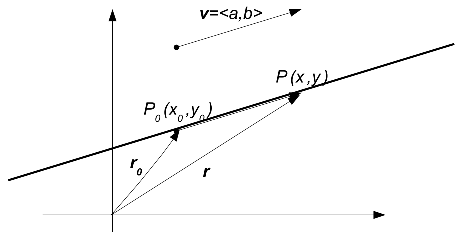
2. At what time is the object at the point (1,8).

3. Does the object pass through the point (3,20).

4. Find an equation in x and y whose graph is the path of the object.

Vector equation of a line.

A line L is determined by a point P_0 on L and a direction. Let $\mathbf{v}$ be a vector parallel to line L. Let P be be an arbitrary point on L and let $\mathbf{r}_0$ and $\mathbf{r}$ be the position vectors of P and P_0.



Then the **vector equation** of line L is

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$$

If $\mathbf{r} =< x(t), y(t) >$, $\mathbf{v} =< a, b >$ and $P(x_0, y_0)$ then **parametric equations** of the line L are

$$x(t) = x_0 + at, \qquad y(t) = y_0 + bt$$

Example 3. Find a vector, parametric, and Cartesian equations for the line containing the point (2,-1) and parallel to $2\mathbf{i} + 3\mathbf{j}$.

Example 4. Find a vector and parametric equations for the line passing through the points $A(1, 3)$ and $B(2, -1)$.