MATH 151, sections 819-821
Fall 2006
SAMPLE OF FINAL EXAM

1. Given vectors $\vec{a} = \vec{\imath} - 2\vec{\jmath}$, $\vec{b} =< -2, 3 >$. Find
(a) a unit vector that has the same direction as $\vec{b}$.
(b) $\text{comp}_{\vec{b}}\vec{a}$, $\text{proj}_{\vec{b}}\vec{a}$.
(c) Let $\vec{c}$ be the vector obtained by rotating $\vec{a}$ by an angle of 60 degrees in the counterclockwise direction. Compute the vector $\vec{c}$.

2. Find the limits (do not use the L'Hospitale's Rule).
(a) $\lim\limits_{x\to-\infty}(\sqrt{x^2+x+1} - \sqrt{x^2-x})$.
(b) $\lim\limits_{x\to 0} \frac{\sin x + \sin 2x}{\sin 3x}$.

3. Find the vertical and horizonal asymptotes of the function $y = \frac{x^2+4}{x^2-1}$.

4. Find a vector equation and parametric equation for the line passing through the points (1,-3) and (-2,4). Find the distance from the point (1,1) to the given lane.

5. Find $\frac{dy}{dx}$ for each function
(a) $y = (\sin x)^{x^2}$.
(b) $y(t) = \sin^{-1} t$, $x(t) = \cos^{-1}(t^2)$.
(c) $2x^2 + 2xy + y^2 = x$.

6. Find the quadratic approximation of $1/x$ for x near 4.

7. Determine when the function $f(x) = e^{2x-x^2}$ is increasing, decreasing, concave up, concave down.

8. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

9. Use the L'Hospitale's Rule to find the limits:
(a) $\lim\limits_{x\to\infty} x^{\frac{1}{x}}$.
(b) $\lim\limits_{x\to 0} \left(\frac{1}{x} - \csc x\right)$.

10. Find the area under the curve $y = \sqrt{x}$ above the x-axis between 0 and 4.

11. Evaluate the indefinite integral $\int t^2 \cos(1-t^3)\, dt$.

12. Find the vector function $\vec{r}(t)$ that gives the position of a particle at time t having the acceleration $\vec{a}(t) = 2t\vec{\imath} + \vec{\jmath}$, initial velocity $\vec{v}(t) = \vec{\imath} - \vec{\jmath}$, an initial position $\vec{v}(t) = \vec{\imath}$.

13. Let $f(x) = \sqrt{x^2-1}$ and $g = f^{-1}$. Find
(a) the domain and the range for g
(b) g'.