MATH 151, sections 819-821
Fall 2006
SAMPLE OF FINAL EXAM (SOLUTIONS)

1. (a) $\vec{u} = \left\langle \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$
(b) $\text{comp}_{\vec{b}}\vec{a} = -\frac{8}{\sqrt{13}}$, $\text{proj}_{\vec{b}}\vec{a} = \left\langle \frac{16}{13}, -\frac{24}{13} \right\rangle$.
(c) $\vec{c} =< \frac{1}{2} + \sqrt{3}, \frac{\sqrt{3}}{2} - 1 >$.

2. (a) -1.
(b) 1.

3. $x = 1$, $x = -1$ – vertical asymptotes; $y = 1$ – horizonal asymptote.

4. $\vec{r}(t) = -(2 + 3t)\vec{\imath} + (4 + 7y)\vec{\jmath}$ – the vector equation,
$x(t) = -2 - 3t$, $y(t) = 4 + 7t$ – the parametric equation,
The distance from the point (1,1) to the lane is equal to $\frac{12}{\sqrt{58}}$.

5. (a) $\frac{dy}{dx} = 2x(\sin x)^{x^2} \ln \sin x + x^2 \cos x (\sin x)^{x^2-1}$.
(b) $\frac{dy}{dx} = -\frac{\sqrt{1+t^2}}{2t}$.
(c) $\frac{dy}{dx} = \frac{1-4x-2y}{2x+2y}$.

6. $\frac{1}{x} = \frac{1}{4}\left(1 - \frac{x-4}{4} + \frac{(x-4)^2}{16}\right)$.

7. f is decreasing for $x \in (1, +\infty)$, f is increasing for $x \in (-\infty, 1)$,
$x = 1$ is a point of local maximum,
f is CU for $x \in (-\infty, 2 - \sqrt{2}) \cup (2 + \sqrt{2}, +\infty)$,
f is CD for $x \in (2 - \sqrt{2}, 2 + \sqrt{2})$, $x = 2 - \sqrt{2}$, $x = 2 + \sqrt{2}$ are points of inflection.

8. The length of the side of the base = 40 cm, the height of the box = 20 cm.

9. (a) 1.
(b) 0.

10. $\frac{16}{3}$.

11. $-\frac{\sin(1-t^3)}{3} + C$.

12. $\vec{r}(t) = (\frac{t^3}{3} + t + 1)\vec{\imath} + (\frac{t^2}{2} - t)\vec{\jmath}$.

13. (a) $D(g) = R(f) : x \in [0, +\infty)$, $R(g) = D(f) : y \in (-\infty, -1] \cup [1, +\infty)$.
(b) $g'(x) = \frac{x}{x^2+1}$.