Math 151, 510-512, Spring 2008

Review before Test 3.

04/22/2008

Table of derivatives

1. $(C)' = 0$, C is a constant,
2. $(x)' = 1$,
3. $(x^2)' = 2x$,
4. $(x^n)' = nx^{n-1}$,
5. $(\ln x)' = \frac{1}{x}$,
6. $(\log_a x)' = \frac{1}{x \ln a}$,
7. $(e^x)' = e^x$,
8. $(a^x)' = a^x \ln a$,
9. $(\sin x)' = \cos x$,
10. $(\cos x)' = -\sin x$,
11. $(\tan x)' = \sec^2 x$,
12. $(\cot x)' = -\csc^2 x$,
13. $(\sec x)' = \sec x \tan x$,
14. $(\csc x)' = -\csc x \cot x$,
15. $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$,
16. $(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$,
17. $(\tan^{-1} x)' = \frac{1}{1+x^2}$,
18. $(\cot^{-1} x)' = -\frac{1}{1+x^2}$,
19. $(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$,
20. $(\csc^{-1} x)' = -\frac{1}{x\sqrt{x^2-1}}$.

Section 4.3 **Logarithmic functions**

$$\boxed{\log_a x = y \iff a^y = x}$$

The cancellation equations

$$\boxed{\log_a a^x = x} \qquad \boxed{a^{\log_a x} = x}$$

Theorem Function $f(x) = \log_a x$ is one-to-one continuous function with domain $(0, \infty)$ and range $(-\infty, \infty)$.

If $a > 1$, then

$f(x) = \log_a x$ is increasing function, $\boxed{\lim_{x \to \infty} \log_a x = \infty}$,

$$\boxed{\lim_{x \to 0^+} \log_a x = -\infty}$$

if $0 < a < 1$, then

$f(x) = \log_a x$ is decreasing function, $\boxed{\lim_{x \to \infty} \log_a x = -\infty}$,

$$\boxed{\lim_{x \to 0^+} \log_a x = \infty}$$

$\boxed{\log_e x = \ln x}$

If $x, y > 0$ and k is a constant, then

1. $\boxed{\log_a xy = \log_a x + \log_a y}$
2. $\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$
3. $\boxed{\log_a x^k = k \log_a x}$ $\boxed{\log_a \left(\frac{1}{x}\right) = -\log_a x}$
4. $\boxed{\log_{a^k} x = \frac{1}{k} \log_a x}$ $\boxed{\log_{\frac{1}{a}} x = -\log_a x}$
5. $\boxed{\log_a a = 1}$
6. $\boxed{\log_a 1 = 0}$
7. $\boxed{\log_a x = \dfrac{\log_b x}{\log_b a}}$

Example (a) Solve the equation $\log_2(2x+3) = 3$

$$\boxed{\log_e x = \ln x}$$

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1. $\boxed{\log_a xy = \log_a x + \log_a y}$
2. $\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$
3. $\boxed{\log_a x^k = k \log_a x}$ $\qquad$ $\boxed{\log_a \left(\frac{1}{x}\right) = -\log_a x}$
4. $\boxed{\log_{a^k} x = \frac{1}{k}\log_a x}$ $\qquad$ $\boxed{\log_{\frac{1}{a}} x = -\log_a x}$
5. $\boxed{\log_a a = 1}$
6. $\boxed{\log_a 1 = 0}$
7. $\boxed{\log_a x = \dfrac{\log_b x}{\log_b a}}$

Example (a) Solve the equation $\log_2(2x + 3) = 3$
(b) Find the solution to the equation $2^x + 3 \cdot 2^x = 24$

Section 4.4 **Derivatives of logarithmic functions**

$$\boxed{(\ln x)' = \frac{1}{x}}$$

$$\boxed{(\ln(g(x))' = \frac{g'(x)}{g(x)}}$$

Example Find the derivative to the function
$f(x) = \ln(\ln(\sin^{-1}(x^2)))$

Section 4.4 **Derivatives of logarithmic functions**

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$$\boxed{(\ln(g(x))' = \frac{g'(x)}{g(x)}}$$

Example Find the derivative to the function $f(x) = \ln(\ln(\sin^{-1}(x^2)))$

Logarithmic differentiation

Steps in logarithmic differentiation

1. Take the logarithm of both sides of an equation.
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y'.

Example 7. Differentiate each function

Section 4.4 **Derivatives of logarithmic functions**

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Logarithmic differentiation

Steps in logarithmic differentiation

1. Take the logarithm of both sides of an equation.
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y'.

Example 7. Differentiate each function
(a) $f(x) = \frac{(x-2)^3(3x-1)^{\frac{1}{3}}}{2\sqrt{x+1}}$

Section 4.4 **Derivatives of logarithmic functions**

$$\boxed{(\ln x)' = \frac{1}{x}}$$

$$\boxed{(\ln(g(x))' = \frac{g'(x)}{g(x)}}$$

Example Find the derivative to the function
$f(x) = \ln(\ln(\sin^{-1}(x^2)))$

Logarithmic differentiation

Steps in logarithmic differentiation

1. Take the logarithm of both sides of an equation.
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y'.

Example 7. Differentiate each function

(a) $f(x) = \frac{(x-2)^3(3x-1)^{\frac{1}{3}}}{2\sqrt{x+1}}$ (b) $f(x) = (x + x^2)^{\tan x}$

Section 4.5 **Exponential growth and decay**

If $y(t)$ is the value of a quantity y at time t and if the rate of change of y with respect to t is proportional to $y(t)$ at any time, then $\boxed{\dfrac{dy}{dt} = ky}$ where k is a constant. This equation is called the **law of natural growth** if $k > 0$ or the the **law of natural decay** if $k < 0$.

The only solution to this equation is $\boxed{y(t) = y(0)e^{kt}}$

Example Polonium-210 has a half-life of 140 days. If a sample has a mass of 200 mg, find the mass after 100 days.

Section 4.6 **Inverse trigonometric functions**

Inverse sine function $\boxed{\arcsin x = \sin^{-1} x = y \quad \Leftrightarrow \quad \sin y = x}$

DOMAIN $\quad -1 \leq x \leq 1$

RANGE $\quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

CANCELLATION EQUATIONS

$$\boxed{\sin^{-1}(\sin x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}}$$

$$\boxed{\sin(\sin^{-1} x) = x \quad \text{for } -1 \leq x \leq 1}$$

$$\boxed{(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}}$$

Inverse cosine function

$$\boxed{\arccos x = \cos^{-1} x = y \quad \Leftrightarrow \quad \cos y = x}$$

DOMAIN $\quad -1 \leq x \leq 1$

RANGE $\quad 0 \leq y \leq \pi$

CANCELLATION EQUATIONS

$$\boxed{\cos^{-1}(\cos x) = x \quad \text{for } 0 \leq x \leq \pi}$$

$$\boxed{\cos(\cos^{-1} x) = x \quad \text{for } -1 \leq x \leq 1}$$

$$\boxed{(\cos^{-1} x)' = -\frac{1}{\sqrt{1 - x^2}}}$$

Inverse tangent function

$$\boxed{\arctan x = \tan^{-1} x = y \quad \Leftrightarrow \quad \tan y = x}$$

DOMAIN $\quad -\infty \leq x \leq \infty$

RANGE $\quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

CANCELLATION EQUATIONS

$$\boxed{\tan^{-1}(\tan x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}}$$

$$\boxed{\tan(\tan^{-1} x) = x \quad \text{for } -\infty \leq x \leq \infty}$$

$$\boxed{\lim_{x\to-\infty} \tan^{-1} x = -\frac{\pi}{2}}$$

$$\boxed{\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}}$$

$$\boxed{(\tan^{-1} x)' = \frac{1}{1+x^2}}$$

Inverse cotangent function

$$\boxed{\operatorname{arccot} x = \cot^{-1} x = y \quad \Leftrightarrow \quad \cot y = x}$$

DOMAIN $\quad -\infty \leq x \leq \infty$

RANGE $\quad 0 < y < \pi$

CANCELLATION EQUATIONS

$$\boxed{\cot^{-1}(\cot x) = x \quad \text{for } 0 \leq x \leq \pi}$$

$$\boxed{\cot(\cot^{-1} x) = x \quad \text{for } -\infty \leq x \leq \infty}$$

$$\boxed{\lim_{x \to -\infty} \cot^{-1} x = 0}$$

$$\boxed{\lim_{x \to \infty} \cot^{-1} x = \pi}$$

$$\boxed{(\cot^{-1} x)' = -\frac{1}{1 + x^2}}$$

Other inverse trigonometric functions

$$\boxed{\csc^{-1} x = y \quad \Leftrightarrow \quad \csc y = x}$$

DOMAIN $\quad |x| \geq 1$

RANGE $\quad y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$

$$\boxed{(\csc^{-1} x)' = -\frac{1}{x\sqrt{x^2 - 1}}}$$

$$\boxed{\sec^{-1} x = y \quad \Leftrightarrow \quad \sec y = x}$$

DOMAIN $\quad |x| \geq 1$

RANGE $\quad y \in \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$

$$\boxed{(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2 - 1}}}$$

Example Simplify $\sin(\tan^{-1} x)$

Other inverse trigonometric functions

$$\boxed{\csc^{-1} x = y \quad \Leftrightarrow \quad \csc y = x}$$

DOMAIN $\quad |x| \geq 1$

RANGE $\quad y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$

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DOMAIN $\quad |x| \geq 1$

RANGE $\quad y \in \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$

$$\boxed{(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2 - 1}}}$$

Example Simplify $\sin(\tan^{-1} x)$

Example Find the derivative of the function
$f(x) = \sin^{-1}(\arctan(2x^2 + 3))$

Section 4.8 **Indeterminate forms and L'Hospitale's rule**

L'Hospital's Rule Suppose f and g are differentiable and $g'(x) \neq 0$ for points close to a (except, possibly a). Suppose that $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$ or $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = \infty$. Then

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \left| \frac{0}{0} \text{ or } \frac{\infty}{\infty} \right| = \lim_{x \to a} \frac{f'(x)}{g'(x)}}$$

Indeterminate products If $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = 0$ or $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = \infty$, then

$$\lim_{x \to a} f(x)g(x) = |\infty \cdot 0| = \lim_{x \to a} \frac{f(x)}{1/g(x)} = \lim_{x \to a} \frac{g(x)}{1/f(x)} = \left| \frac{0}{0} \text{ or } \frac{\infty}{\infty} \right|$$

and now we can use L'Hospital's Rule.

Indeterminate differences If we have to find $\lim_{x\to a}(f(x)-g(x)) = |\infty-\infty|$ $(\lim_{x\to a} f(x)=\infty, \lim_{x\to a} g(x)=\infty)$, then we have to convert the difference into a quotient (by using a common denominator or rationalization, or factoring out a common factor) so that we have an indeterminate form of type $\left|\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right|$ and we can use L'Hospital's Rule.

Indeterminate powers $\lim_{x\to a}[f(x)]^{g(x)} = |0^0 \text{ or } \infty^0 \text{ or } 1^\infty| = \lim_{x\to a} e^{g(x)\ln f(x)} = e^{\lim_{x\to a}[g(x)\ln f(x)]}$. Now let's find

$$\lim_{x\to a}[g(x)\ln f(x)] = |0\cdot\infty| = \lim_{x\to a}\frac{\ln f(x)}{\frac{1}{g(x)}} = \left|\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right| = b$$

Then
$\lim_{x\to a}[f(x)]^{g(x)} = e^b$

Example Evaluate each limit:

Indeterminate differences If we have to find $\lim_{x\to a}(f(x)-g(x)) = |\infty-\infty|$ ($\lim_{x\to a} f(x)=\infty, \lim_{x\to a} g(x)=\infty$), then we have to convert the difference into a quotient (by using a common denominator or rationalization, or factoring out a common factor) so that we have an indeterminate form of type $\left|\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right|$ and we can use L'Hospital's Rule.

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Then
$\lim_{x\to a}[f(x)]^{g(x)} = e^b$

Example Evaluate each limit:
(a) $\lim_{x\to\infty}\frac{\ln x}{\sqrt[3]{x}}$

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Example Evaluate each limit:

(a) $\lim_{x\to\infty}\frac{\ln x}{\sqrt[3]{x}}$ (b) $\lim_{x\to 0}(1-\cos x)\cot x$

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Example Evaluate each limit:

(a) $\lim_{x\to\infty}\frac{\ln x}{\sqrt[3]{x}}$ (b) $\lim_{x\to 0}(1-\cos x)\cot x$

(c) $\lim_{x\to\infty}(x-\sqrt{x^2-1})$

Indeterminate differences If we have to find $\lim_{x\to a}(f(x)-g(x)) = |\infty - \infty|$ ($\lim_{x\to a} f(x)=\infty, \lim_{x\to a} g(x)=\infty$), then we have to convert the difference into a quotient (by using a common denominator or rationalization, or factoring out a common factor) so that we have an indeterminate form of type $\left|\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right|$ and we can use L'Hospital's Rule.

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Example Evaluate each limit:

(a) $\lim_{x\to\infty}\frac{\ln x}{\sqrt[3]{x}}$ (b) $\lim_{x\to 0}(1-\cos x)\cot x$

(c) $\lim_{x\to\infty}(x-\sqrt{x^2-1})$ (d) $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$

Section 5.1 **What does f' say about f?**

If $f'(x) > 0$ on an interval, then f is increasing on that interval

If $f'(x) < 0$ on an interval, then f is decreasing on that interval

f has a **local maximum** at the point, where its derivative changes from positive to negative.

f has a **local minimum** at the point, where its derivative changes from negative to positive.

What does f'' say about f?

If $f''(x) > 0$ on an interval, then f is **concave upward (CU)** on that interval

If $f''(x) < 0$ on an interval, then f is **concave downward (CD)** on that interval

A point where curve changes its direction of concavity is called an **inflection point**

Section 5.2 **Maximum and minimum values**

Definition A function f has an **absolute maximum** or (**global maximum**) at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f. The number $f(c)$ is called the **maximum value** of f on D. Similarly, f has an **absolute minimum** or **global minimum** at c if $f(c) \leq f(x)$ for all x in D and the number $f(c)$ is called the **minimum value** of f on D. The maximum and the minimum values of f are called the **extreme values** of f.

Definition A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c. [This means that $f(c) \geq f(x)$ for all x in some *open* interval containing c]. Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c.

The extreme value theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Fermat's theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$

Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

If f has a local extremum at c, then c is a critical number of f.

The closed interval method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:
(a) Find the values of f at the critical numbers of f in (a, b)
(b) Find $f(a)$ and $f(b)$
(c) The largest number of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Example Find the absolute maximum and absolute minimum values of $f(x) = x^3 - 2x^2 + x$ on [-1,1].

Section 5.3 **Derivatives and the shapes of curves.**

The mean value theorem If f is a differentiable function on the interval $[a, b]$, then there exist a number c, $a < c < b$, such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}} \text{ or } \boxed{f(b) - f(a) = f'(c)(b - a)}.$$

Increasing / decreasing test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval

The first derivative test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local max at c.

(b) If f' changes from negative to positive at c, then f has a local min at c.

(c) If f' does not change sign c, then f has a no local max or min at c.

A function is called *concave upward* (CU) on an interval I if f' is an increasing function on I. It is called *concave downward* (CD) on I if f' is an decreasing on I.

A point where a curve changes its direction of concavity is called an *inflection point*.

Concavity test

(a) If $f''(x) > 0$ on an interval, then f is CU on this interval.

(b) If $f''(x) < 0$ on an interval, then f is CD on this interval.

The second derivative test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local min at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local max at c.

Example Sketch the graph of the function $f(x) = x^3 - 3x^2 + 3$

Section 5.5 **Applied maximum and minimum problems**

Steps in solving applied max and min problems

1. Understand the problem.

2. Draw a diagram.

3. Introduce notation. Assign a symbol to the quantity that is to be minimized or maximized (let us call it Q). Also select symbols $(a, b, c, \ldots, x, y)$ for other unknown quantities and label the diagram with these symbols.

4. Express Q in terms of some of the other symbols from step 3.

5. If Q has been expressed as a function of more than one variable in step 4, use the given information to find relationships (in the form of equation) among these variables. Then use these equations to eliminate all but one of the variables in the expression for Q. Thus, Q will be given as a function of one variable.

6. Find the *absolute* maximum or minimum of Q.

Example Rectangular box with open top has height h, length l and width w. The length of the box is twice its width and the volume of the box is 9 ft^3 The material for the base costs \$10 per ft^2 and the material for thee sides costs \$5 per ft^2. Find the dimension of the box that will minimize the cost of the material.

Section 5.7 **Antiderivatives**

Definition Function $F(x)$ is called an **antiderivative** of $f(x)$ on an interval I if $F'(x) = f(x)$ for all $x \in I$.

Theorem If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $\boxed{F(x) + C}$ where C is a constant.

Table of antidifferentiation formulas

Function	*Antiderivative*
$af(x)$, a is a constant	$aF(x) + C$
$f(x) + g(x)$	$F(x) + G(x) + C$
a, a is a constant	$ax + C$
x	$\frac{x^2}{2} + C$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln\|x\| + C$

Function	*Antiderivative*
e^x	$e^x + C$
a^x	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} + C$

Example Find the most general antiderivative of the function

Function	*Antiderivative*
e^x	$e^x + C$
a^x	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
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$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} + C$

Example Find the most general antiderivative of the function
(a) $f(x) = (\sqrt{x} + 1)(x - \sqrt{x} + 1)$

Function	*Antiderivative*
e^x	$e^x + C$
a^x	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$
$\csc^2 x$	$-\cot x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} + C$

Example Find the most general antiderivative of the function
(a) $f(x) = (\sqrt{x} + 1)(x - \sqrt{x} + 1)$
(b) $f(x) = \sin t + \frac{2}{1+x^2} + \frac{3}{\sqrt{1-x^2}}$

Rectilinear motion

If the object has a position function $s = s(t)$, then $v(t) = s'(t)$ (the position function is an antiderivative for the velocity function), $a(t) = v'(t)$ (the velocity function is an antiderivative to the acceleration function)

Example A particle is moving with the acceleration $a(t) = t^2 - t$, $s(0) = 0$, $v(0) = 1$. Find the position of the particle.

Rectilinear motion

If the object has a position function $s = s(t)$, then $v(t) = s'(t)$ (the position function is an antiderivative for the velocity function), $a(t) = v'(t)$ (the velocity function is an antiderivative to the acceleration function)

Example A particle is moving with the acceleration $a(t) = t^2 - t$, $s(0) = 0$, $v(0) = 1$. Find the position of the particle.

Antiderivatives of vector functions

Definition A vector function $\vec{R}(t) =< X(t), Y(t) >$ is called **an antiderivative** of $\vec{r}(t) =< x(t), y(t) >$ on an interval I if $\vec{R}'(t) = \vec{r}(t)$ that is, $X'(t) = x(t)$ and $Y'(t) = y(t)$.

Theorem If $\vec{R}$ is an antiderivative of $\vec{r}$ on an interval I, then the most general antiderivative of $\vec{r}$ on I is $\boxed{\vec{R} + \vec{C}}$ where $\vec{C}$ is an arbitrary constant vector.

Example Find the vector-function that describe the position of particle that has an acceleration $\vec{a}(t) = 2t\vec{\imath} + 3\vec{\jmath}$, $\vec{v}(0) = \vec{\imath} - \vec{\jmath}$, and initial position at (1,2).

Section 6.1 **Sigma notation**

Definition If a_m, a_{m+1}, a_{m+2},...,a_n are real numbers and m and n are integers such that $m \leq n$, then

$$\boxed{a_m + a_{m+1} + a_{m+2} + \ldots + a_n = \sum_{i=m}^{n} a_i}$$

Theorem If c is any constant (this means that c does not depend on i), then

(a) $\displaystyle\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$ (b) $\displaystyle\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

$$\boxed{\sum_{i=1}^{n} 1 = n} \qquad \boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}} \qquad \boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$\boxed{\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2} \qquad \boxed{\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}}$$

Example Find the value of the sum $\displaystyle\sum_{i=0}^{5} i(i-1)$.

Section 6.2 **Area**

Problem Find the area of the region S that lies under the curve $y = f(x)$ from a to b.
Let P be a partition of $[a, b]$ with partition points x_0, x_1,...,x_n, where

$$a = x_0 < x_1 < x_2 < ... < x_n = b$$

Choose points $x_i^* \in [x_{i-1}, x_i]$ and let $\Delta x_i = x_i - x_{i-1}$ and $\|P\| = \max\{\Delta x_i\}$.
Then area of S is

$$\boxed{A = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i}$$

Example Find the area under the curve $y = 1 + x^3$ above the x-axis between $x = 2$ and $x = 6$. Use eight subintervals of equal length and take x_i^* to be the right endpoint of the ith subinterval.