# Math 152, Fall 2008

## Lecture 1.

08/26/2008

# Table of indefinite integrals

1. $\int a dx = ax + C,\ \ a$ is a constant,
2. $\int x dx = \frac{x^2}{2} + C$,
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$,
4. $\int \frac{1}{x} dx = \ln|x| + C$,
5. $\int e^x dx = e^x + C$,
6. $\int a^x dx = \frac{a^x}{\ln a} + C$,
7. $\int \sin x dx = -\cos x + C$,
8. $\int \cos x dx = \sin x + C$,
9. $\int \tan x dx = -\ln|\cos x| + C$,
10. $\int \cot x dx = \ln|\sin x| + C$,
11. $\int \sec^2 x dx = \tan x + C$,
12. $\int \csc^2 x dx = -\cot x + C$,
13. $\int \sec x \tan x dx = \sec x + C$,
14. $\int \csc x \cot x = -\csc x + C$,
15. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$,
16. $\int \frac{1}{1+x^2} dx = \arctan x + C$.

# Definition of a definite integral

If $f$ is a function defined on a closed interval $[a, b]$, let $P$ be a partition of $[a, b]$ with partition points $x_0$, $x_1$,...,$x_n$, where

$$a = x_0 < x_1 < x_2 < \ldots < x_n = b$$

Choose points $x_i^* \in [x_{i-1}, x_i]$ and let $\Delta x_i = x_i - x_{i-1}$ and $\|P\| = \max\{\Delta x_i\}$. Then the **definite integral of $f$ from $a$ to $b$** is

$$\boxed{\int_a^b f(x)dx = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i}$$

if this limit exists. If the limit does exist, then $f$ is called **integrable** on the interval $[a, b]$.

In the notation $\int_a^b f(x)dx$, $f(x)$ is called the **integrand** and $a$ and $b$ are called the limits of integration; $a$ is the **lower limit** and $b$ is the **upper limit**.

The procedure of calculating an integral is called **integration**.

# Properties of the definite integral

1. $\int_a^b c dx = c(b-a)$, where $c$ is a constant.
2. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, where $c$ is a constant.
3. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$.
4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$.
5. $\int_a^b f(x)dx = -\int_b^a f(x)dx$.
6. If $f(x) \geq 0$ for $a < x < b$, then $\int_a^b f(x)dx \geq 0$.
7. If $f(x) \geq g(x)$ for $a < x < b$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

8. If $m \le f(x) \le M$ for $a < x < b$, then
$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$.
9. $\left| \int_a^b f(x)dx \right| \le \int_a^b |f(x)|dx$

Section 6.4 **The fundamental theorem of calculus**

Suppose $f$ is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$.
2. $\boxed{\int_a^b f(x)dx = F(b) - F(a)}$, where $F$ is an antiderivative of $f$.

**Example 1.** Evaluate the integral.

1. $\int (6x^2 + 8x + 3)dx$
2. $\int_0^8 (\sqrt{2x} + \sqrt[3]{x})dx$
3. $\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}}dx$
4. $\int_2^6 \frac{1+\sqrt{y}}{y^2}dy$
5. $\int_0^2 f(x)dx$, where $f(x) = \begin{cases} x^4 & 0 \le x < 1 \\ x^5 & 1 \le x \le 2 \end{cases}$
6. $\int_{-1}^2 (x - 2|x|)dx$

Section 6.5 **The Substitution Rule**

**The substitution rule for indefinite integrals** If $u = g(x)$ is a differentiable function whose range is an interval $I$ and $f$ is continuous on $I$, then $\boxed{\int f(g(x))g'(x)dx = \int f(u)du}$.

**The substitution rule for definite integrals** If $g'(x)$ is continuous on $[a, b]$ and $f$ is continuous on the range of $g$, then

$$\boxed{\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du}.$$

If $F(x)$ is an antiderivative to $f(x)$, then

$$\boxed{\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C}$$

**Integrals of symmetric functions** Suppose $f$ is continuous on $[-a, a]$.

(a) If $f$ is **even**, then $\boxed{\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx}$

(b) If $f$ is **odd**, then $\boxed{\int_{-a}^{a} f(x)dx = 0}$

**Example 2.** Evaluate each integral.

(a) $\int \frac{dx}{\sqrt{5x-2}}$ (b) $\int \frac{xdx}{\sqrt{1+x^4}}$ (c) $\int x^2 e^{x^3} dx$

(d) $\int_{1}^{4} \frac{1}{x^2}\sqrt{1+\frac{1}{x}}dx$ (e) $\int_{-2}^{2} x\sqrt{x^2+a^2}dx$ (f) $\int_{e}^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(g) $\int \frac{\sin x}{1+\cos^2 x}dx$ (h) $\int t^2\cos(1-t^3)dt$ (i) $\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6}dx$

(j) $\displaystyle\int \frac{dx}{x^2 + a^2}$

(k) $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}}$

(l) $\displaystyle\int \frac{\arcsin x + x}{\sqrt{1 - x^2}} dx$