Math 152, Fall 2008

Lecture 2.

08/28/2008

Example 1. Evaluate each integral.

(a) $\displaystyle\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$

(b) $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(c) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

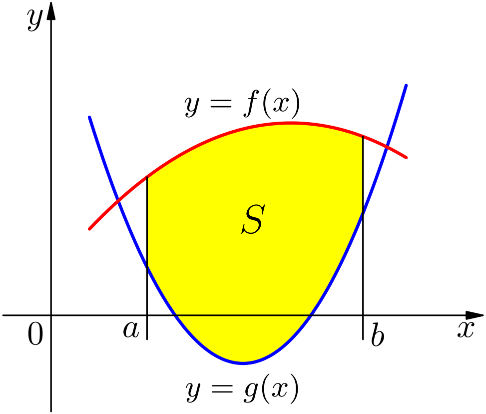
(d) $\int t^2\cos(1-t^3)\,dt$

(e) $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x}{1+x^6}\,dx$

(f) $\displaystyle\int \frac{dx}{x^2+a^2}$

(j) $\displaystyle\int \frac{\arcsin x + x}{\sqrt{1-x^2}}\,dx$

Consider the region S that lies between two curves $y = f(x)$ and $y = g(x)$ and between the vertical lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a, b]$.



$$S = \{(x, y) : a \leq x \leq b, g(x) \leq y \leq f(x)\}$$

Let P be a partition of $[a, b]$ by points x_i and choose points x_i^* in $[x_{i-1}, x_i]$. Let $\Delta x_i = x_i - x_{i-1}$ and $\|P\| = \max\{\Delta x_i\}$.

The Riemann sum $\sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\Delta x_i$ is an approximation to what we intuitively think as the area of S.

This approximation appears to become better and better as $\|P\| \to 0$. We define the area A of S as

$$\boxed{A = \lim_{\|P\| \to 0} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\Delta x_i}$$

Therefore:

The area of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a, b]$, is

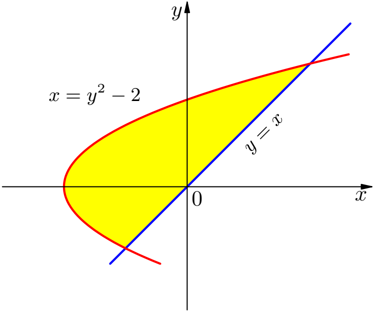
$$\boxed{A = \int_a^b [f(x) - g(x)]dx}$$

Example 2. Find the area of the region bounded by the parabolas $y = 4x^2$, $y = x^2 + 3$

In general case, the area between the curves $y = f(x)$, $y = g(x)$ and between $x = a$ and $x = b$, is $\boxed{A = \int_a^b |f(x) - g(x)| dx}$

Example 3. Find the area of the region bounded by the curves $y = \cos x$, $y = \sin 2x$ an the lines $x = 0$ and $x = \pi/2$.

Example 4. Find the area of the shaded region.



If a region is bounded by curves with equations $x = f(y)$, $x = g(y)$, $y = c$ and $y = d$, where f and g are continuous functions and $f(y) \geq g(y)$ for all y in $[c, d]$, then its area is

$$A = \int_c^d [f(y) - g(y)] dy$$