Math 152, Fall 2008

Lecture 6.

09/11/2008

Help Sessions:

Sunday	7:30PM – 9:30PM	BLOC 120
Monday	7:30PM – 9:30PM	BLOC 155
Wednesday	8:00PM – 10:00PM	BLOC 120

HW#3 is due Wednesday, September 17, 11:55 PM.

Chapter 8. **Techniques of integration**
Section 8.1 **Integration by parts**

Integration by parts is a rule that transforms the integral of products of functions into other, hopefully simpler, integral. Assume that f and g are continuous functions.

The **formula for integration by parts for indefinite integrals** is

$$\boxed{\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx}$$

The **formula for integration by parts for definite integrals** is

$$\boxed{\int_a^b f(x)g'(x)dx = f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx}$$

Integration by parts is a heuristic rather than a purely mechanical process for solving integrals; given a single function to integrate, the **typical strategy** is to carefully separate it into a product of two functions $f(x)g(x)$ such that the integral produced by the integration by parts formula is easier to evaluate than the original one.

Example 1. Evaluate the integral.

(a) $\int \ln x \; dx$ (b) $\int \arcsin x \; dx$ (c) $\int x \cos 3x \; dx$

(d) $\int_0^1 t^2 e^t dt$ (e) $\int \cos(\ln x) \; dx$ (f) $\int e^x \cos x \; dx$

Example 2. (a) Prove the reduction formula

$$\int \cos^n x \; dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \; dx.$$

(b) evaluate integral $\int \cos^4 x \; dx$

Example 3. Use the methods of cylindrical shells to find the volume generated by rotating the region bounded by $y = e^{-x}$, $y = 0$, $x = -1$, $x = 0$ about $x = 1$.

Section 8.2 **Trigonometric integrals**

How to evaluate $\int \sin^m x \cos^n x \ dx$

(a) if $n = 2k+1$ (n is odd), save one cosine factor and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine:

$$\int \sin^m x \cos^{2k+1} x \ dx = \int \sin^m x (\cos^2 x)^k \cos x \ dx =$$

$$\int \sin^m x (1 - \sin^2 x)^k \cos x \ dx$$

Then substitute $u = \sin x$

(b) if $m = 2s + 1$ (m is odd), save one sine factor and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine:

$$\int \sin^{2s+1} x \cos^n x \; dx = \int (\sin^2 x)^s \cos^n x \sin x \; dx =$$

$$\int (1 - \cos^2 x)^s \cos^n x \sin x \; dx$$

Then substitute $u = \cos x$

(c) if both m and n are even, use the half-angle identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

It is sometimes useful to use the identity

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

Example 4. Evaluate each of the following integrals

(a) $\int \cos^3 x \ dx$ (b) $\int \sin^5 x \ dx$ (c) $\int\limits_0^{\pi/2} \sin^2 3x \ dx$

(d) $\int \sin^2 x \cos^3 x \ dx$ (e) $\int\limits_0^{\pi/4} \sin^4 x \cos^3 x \ dx$

(f) $\displaystyle\int \sin^3 \frac{x}{2} \cos^5 \frac{x}{2} \ dx$ (g) $\int\limits_0^{\pi/2} \sin^2 x \cos^2 x \ dx$ (h) $\int \cos^4 x \ dx$