Math 152, Fall 2008

Lecture 8.

09/18/2008

HW#4 is due Wednesday, September 24, 11:55 PM.

Chapter 8. **Techniques of integration**
Section 8.3 **Trigonometric substitution**

Assume that g is one-to-one function (g^{-1} exists). Then $\boxed{\int f(x)dx = \int f(g(t))g'(t)dt}$. This kind of substitution is called *inverse substitution*.

Table of trigonometric substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin t, -\pi/2 \leq t \leq \pi/2$	$1 - \sin^2 t = \cos^2 t$
$\sqrt{a^2 + x^2}$	$x = a\tan t, -\pi/2 < t < \pi/2$	$1 + \tan^2 t = \sec^2 t$
$\sqrt{x^2 - a^2}$	$x = a\sec t, 0 \leq t \leq \pi/2$ or $\pi \leq t \leq 3\pi/2$	$\sec^2 t - 1 = \tan^2 t$

Example 1.

(a) $\int x\sqrt{4-x^2}dx$ (b) $\int \frac{x^3}{\sqrt{x^2+4}}dx$

(c) $\int \frac{dx}{x^2\sqrt{16x^2-9}}$ (d) $\int \frac{dx}{\sqrt{x^2+4x+8}}$

Section 8.4 **Integration of rational functions by partial fractions**

In this section we show how to integrate any rational function $f(x) = \dfrac{P(x)}{Q(x)}$, where $P(x) = a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n$, $Q(x) = b_0x^m + b_1x^{m-1} + ... + b_m$ by expressing it as a sum of *partial fractions*, that we know how to integrate.

STEP 1. If f is improper ($m \geq n$), then we must divide Q into P by long divisions until a remainder $R(x)$ is obtained. The division statement is

$$f(x) = \frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$$

STEP 2. Factor the denominator $Q(x)$ as far as possible. It can be shown that any polynomial Q can be factored as a product of *linear factors* of the form $ax + b$ and *irreducible quadratic factors* (of the form $ax^2 + bx + c$, where $b^2 - 4ac < 0$).

STEP 3. Express the proper rational function $\frac{R(x)}{Q(x)}$ as a sum of **partial fractions** of the form

$$\frac{A}{(ax+b)^i} \quad \text{or} \quad \frac{Ax+B}{(ax^2+bx+c)^j}$$

Four cases occur.

CASE I. $Q(x)$ **is a product of distinct linear factors.**

$$Q(x) = (a_1x + b_1)(a_2x + b_2)...(a_mx + b_m)$$

where no factor is repeated. Then there exist constants A_1, A_2,..., A_m such that

$$f(x) = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + ... + \frac{A_m}{a_mx + b_m}$$

Once the constants A_1, A_2,... , A_m are determined, the evaluation of $\frac{R(x)}{Q(x)}$ becomes a routine problem. The next examples will illustrate one method for finding these constants.

Example 2. Evaluate $\displaystyle\int_{2}^{4} \frac{4x-1}{x^2+x-2}dx$

CASE II. $Q(x)$ **is a product of linear factors, some of which are repeated.**

Suppose the first linear factor $a_1x + b_1$ is repeated r times; that is, $(a_1x + b_1)^r$ occurs in factorization of $Q(x)$. Then instead of the single term $A_1/(a_1x + b_1)$, we would use

$$\frac{A_1}{a_1x+b_1} + \frac{A_2}{(a_1x+b_1)^2} + \ldots + \frac{A_r}{(a_1x+b_1)^r}$$

Example 3. Evaluate $\displaystyle\int \frac{5x^2+6x+9}{(x+1)^2(x-3)^2}dx$