1. Find the area of the region bounded by the parabola $y = x^2$, tangent line to this parabola at $(1, 1)$, and the x-axis.

2. Find the volume of a frustum of a right circular cone with height h, lower base radius R, and top radius r.

3. Use **washers** to find the volume of a solid bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \dfrac{\pi}{2}$ about the line $y = 1$.

4. Use **cylindrical shells** to find the volume of a solid bounded by $y = e^x$, $y = e^{-x}$, $x = 1$ about the y-axis.

5. The tank in a shape of hemisphere with radius 5 ft is full of water. Find the work done in pumping all the water to the top of the tank. Use the fact that water weighs 62.5 lb/ft^3.

6. Evaluate the integral

   (a) $\int \sin^2 x \cos^4 x \, dx$

   (b) $\int_0^{\pi/4} \tan^4 x \sec^2 x \, dx$

   (c) $\int \tan x \sec^3 x \, dx$

   (d) $\int \sin 3x \cos x \, dx$

   (e) $\displaystyle\int \frac{x^2}{\sqrt{5 - x^2}} dx$

   (f) $\displaystyle\int \frac{x^3}{\sqrt{x^2 + 4}} dx$

   (g) $\displaystyle\int \frac{\sqrt{9x^2 - 4}}{x} dx$

   (h) $\displaystyle\int \frac{x^2 dx}{(x - 3)(x + 2)^2}$

   (i) $\displaystyle\int \frac{x^4 dx}{x^4 - 1}$

   (j) $\displaystyle\int \frac{x^4 + 1}{x(x^2 + 1)^2} dx$

   (k) $\displaystyle\int \frac{x dx}{x^2 + x + 1}$

   (l) $\displaystyle\int \frac{x dx}{(x^2 + x + 1)^2}$

7. Determine whether the integral is convergent. Evaluate those that are convergent.

   (a) $\displaystyle\int_0^{\infty} \frac{dx}{(x + 2)(x + 3)}$

(b) $\int_{0}^{\infty} xe^{-x}\, dx$

(c) $\int_{1}^{17} \frac{dx}{\sqrt[3]{x-9}}$

8. Find the length of the curve.

   (a) $y = \ln(\sin x)$, $\pi/6 \le x \le \pi/3$

   (b) $x = y^{3/2}$, $0 \le y \le 1$

   (c) $x = 3t - t^3$, $y = 3t^2$, $0 \le t \le 2$

9. Find the surface area of a torus.

10. Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

    (a) $a_n = \sin n$

    (b) $a_n = \frac{n}{\ln n}$

    (c) $a_n = \frac{\pi^n}{3^n}$

    (d) $a_n = \frac{n}{2n+5}$

    (e) $a_n = \sqrt{n+2} - \sqrt{n-1}$

11. Show that the sequence defined by $a_1 = 1$, $a_{n+1} = 3 - \frac{1}{a_n}$ is increasing and $a_n < 3$ for all n. Find its limit.

12. Find the sum of the series

    (a) $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

    (b) $\sum_{n=3}^{\infty} \frac{1}{n^2-4}$

13. Which of the following series is convergent?

    (a) $\sum_{n=1}^{\infty} \frac{n^2}{n^{5/7}+1}$

    (b) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$

    (c) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

14. Which of the following series is absolutely convergent?

(a) $\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

(b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$

(c) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{\sqrt{n-2}}$

(d) $\sum_{n=0}^{\infty} (-1)^n \frac{2^{2n}}{3^{3n}}$

15. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$.

16. Find the power series representation for the function $f(x) = \ln(1-2x)$ centered at 0.

17. Find the Taylor series for $f(x) = xe^{2x}$ at $x = 2$.

18. Find the Maclaurin series for $f(x) = x\sin(x^3)$.

19. Find the sum of the series

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n x^2}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

20. Evaluate the indefinite integral as a power series $\int e^{x^2} dx$.

21. Find the angle between the vectors $\vec{a} = \vec{\imath} + \vec{\jmath} + 2\vec{k}$ and $\vec{b} = 2\vec{\jmath} - 3\vec{k}$.

22. Find the directional cosines for the vector $\vec{a} = -2\vec{\imath} + 3\vec{\jmath} + \vec{k}$.

23. Find the scalar and the vector projections of the vector $< 2, -3, 1 >$ onto the vector $< 1, 6, -2 >$.

24. Given vectors $\vec{a} =< -2, 3, 4 >$ and $\vec{b} =< 1, 0, 3 >$. Find $\vec{a} \times \vec{b}$.

25. Find the volume of the parallelepiped determined by vectors $\vec{a} =< 1, 0, 6 >$, $\vec{b} =< 2, 3, -8 >$, and $\vec{c} =< 8, -5, 6 >$.

26. Represent the point with Cartesian coordinates $(2\sqrt{3}, -2)$ in terms of polar coordinates.

27. Sketch the curve $r = \sin 5\theta$.