Table of indefinite integrals

1. $\int a dx = ax + C,\ a$ is a constant,
2. $\int x dx = \frac{x^2}{2} + C,$
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1,$
4. $\int \frac{1}{x} dx = \ln|x| + C,$
5. $\int \mathrm{e}^x dx = \mathrm{e}^x + C,$
6. $\int \mathrm{a}^x dx = \frac{\mathrm{a}^x}{\ln \mathrm{a}} + C,$
7. $\int \sin x dx = -\cos x + C,$
8. $\int \cos x dx = \sin x + C,$
9. $\int \tan x dx = -\ln|\cos x| + C,$
10. $\int \cot x dx = \ln|\sin x| + C,$
11. $\int \sec^2 x dx = \tan x + C,$
12. $\int \csc^2 x dx = -\cot x + C,$
13. $\int \sec x \tan x dx = \sec x + C,$
14. $\int \csc x \cot x = -\csc x + C,$
15. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C,$
16. $\int \frac{1}{1+x^2} dx = \arctan x + C.$

Definition of a definite integral

If f is a function defined on a closed interval $[a, b]$, let P be a partition of $[a, b]$ with partition points x_0, x_1,...,x_n, where

$$a = x_0 < x_1 < x_2 < ... < x_n = b$$

Choose points $x_i^* \in [x_{i-1}, x_i]$ and let $\Delta x_i = x_i - x_{i-1}$ and $\|P\| = \max\{\Delta x_i\}$. Then the **definite integral of** f **from** a **to** b is

$$\int_a^b f(x)dx = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

if this limit exists. If the limit does exist, then f is called **integrable** on the interval $[a, b]$.

In the notation $\int_a^b f(x)dx$, $f(x)$ is called the **integrand** and a and b are called the limits of integration; a is the **lower limit** and b is the **upper limit**.

The procedure of calculating an integral is called **integration**.

Properties of the definite integral

1. $\int_a^b c\,dx = c(b-a)$, where c is a constant.

2. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, where c is a constant.

3. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$.

4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$.

5. $\int_a^b f(x)dx = -\int_b^a f(x)dx$.

6. If $f(x) \geq 0$ for $a < x < b$, then $\int_a^b f(x)dx \geq 0$.

7. If $f(x) \geq g(x)$ for $a < x < b$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

8. If $m \leq f(x) \leq M$ for $a < x < b$, then $m(b-a) \leq \int\limits_a^b f(x)dx \leq M(b-a)$.

9. $\left|\int\limits_a^b f(x)dx\right| \leq \int\limits_a^b |f(x)|dx$

Section 6.4 The fundamental theorem of calculus.

Suppose f is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$.

2. $\int_a^b f(x)dx = F(b) - F(a) = F(x)|_a^b$, where F is an antiderivative of f.

Example 1. Find the derivative of the function.

1. $f(x) = \int_x^4 (2 + \sqrt{t})^8 dt$ $= - \int_4^x (2+\sqrt{t})^8 dt$

2. $y = \int_{\tan x}^{17} \sin(t^4)dt$

3. $y = \int_{\sqrt{x}}^{x^3} \sqrt{t}\sin(t)dt$

Example 2. Evaluate the integral.

1. $\displaystyle\int_1^8 \frac{x^2+1}{\sqrt[3]{x}}\,dx$

2. $\displaystyle\int_4^9 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx = \int_4^9 \left(x + 2\sqrt{x}\,\frac{1}{\sqrt{x}} + \frac{1}{x}\right) dx$

$2 \cdot 1$

$$= \int_4^9 \left(x + 2 + \frac{1}{x}\right) dx = \left[\frac{x^2}{2} + 2x + \ln|x|\right]_4^9$$

$$= \frac{9^2}{2} + 2(9) + \ln 9 - \frac{4^2}{2} - 8 - \ln 4$$

$$= \boxed{\frac{65}{2} + 10 + \ln\frac{9}{4}}$$

3. $\int_0^2 (x^2 - |x-1|)dx$

$$|x-1| = \begin{cases} x-1, & \text{if } x-1 \geq 0 \\ -(x-1), & \text{if } x-1 < 0 \end{cases}$$

$$= \begin{cases} x-1, & \text{if } x \geq 1 \\ 1-x, & \text{if } x < 1 \end{cases}$$

$x^2 - (x-1)$

$$x^2 - |x-1| = \begin{cases} x^2 - x + 1, & \text{if } x \geq 1 \\ x^2 + x - 1, & \text{if } x < 1 \end{cases}$$

$x^2 - (-(x-1))$

$$= \int_0^1 (x^2 + x - 1)\,dx + \int_1^2 (x^2 - x + 1)\,dx = \ldots$$

Example 3. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - 2t - 8$.

1. Find the displacement of the particle during the time period $1 \le t \le 6$.

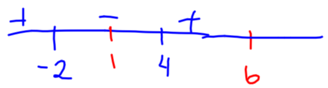
$$\text{displacement} = \int_1^6 v(t)\,dt = \int_1^6 (t^2 - 2t - 8)\,dt = \dots$$

2. Find the distance traveled during this time period.

$$\text{distance travelled} = \int_1^6 |v(t)|\,dt = \int_1^6 |t^2 - 2t - 8|\,dt$$

$$t^2 - 2t - 8 \ge 0$$

$$(t-4)(t+2) \ge 0$$



$$t^2 - 2t - 8 \ge 0 \text{ on } (-\infty, -2] \cup [4, \infty)$$

$$t^2 - 2t - 8 < 0 \text{ on } (-2, 4)$$

$$|t^2 - 2t - 8| = \begin{cases} t^2 - 2t - 8, & \text{if } t \le -2 \text{ or } t \ge 4 \\ -(t^2 - 2t - 8), & \text{if } -2 < t < 4 \end{cases}$$

$$\text{distance} = \int_1^4 \left[-(t^2 - 2t - 8)\right] dt + \int_4^6 (t^2 - 2t - 8)\,dt = \dots$$