## MATH 152, 501-506, Spring 2011, Sample problems for the Final

1. Find the area of the region bounded by $y=x^{2}$ and $y=\sqrt{x}$.
2. Find the volume of the solid obtained by rotating the region bounded by $y=x^{2}-1$, $y=0, x=1, x=2$ about the $x$-axis.
3. Find the volume of the solid obtained by rotating the region bounded by $y=x^{2}, y=0$. $x=1, x=2$ about
(a) the $y$-axis
(b) $x=4$
4. The base of solid $S$ is the triangular region with vertices $(0,0),(2,0)$, and $(0,1)$. Crosssections perpendicular to the $x$-axis are semicircles. Find the volume of $S$.
5. A spring has a natural length of 20 cm . If a $25-\mathrm{N}$ force is required to keep it stretched to a length of 30 cm , how much work is required to stretch it from 20 cm to 25 cm ?
6. Find the average value of $f=\sin ^{2} x \cos x$ on $[-\pi / 2, \pi / 4]$.
7. Evaluate the integral
(a) $\int_{0}^{1} x^{2} e^{-x} d x$
(b) $\int \sin ^{2} x \cos ^{2} x d x$
(c) $\int \sin 3 x \cos x d x$
(d) $\int \tan x \sec ^{3} x d x$
(e) $\int \frac{x^{2}}{\sqrt{5-x^{2}}} d x$
(f) $\int \frac{d x}{x^{2}\left(x^{2}+1\right)}$
(g) $\int_{0}^{\infty} \frac{d x}{(x+2)(x+3)}$
(h) $\int_{2}^{6} \frac{d x}{\sqrt{x-2}}$
8. Find the length of the curve $x(t)=3 t-t^{3}, y(t)=3 t^{2}, 0 \leq t \leq 2$.
9. Find the area of the surface obtained by rotating the curve $y=x^{3}, 0 \leq x \leq 2$ about the $x$-axis.
10. Find the area of the surface obtained by rotating the curve $x=\sqrt{2 y-y^{2}}, 0 \leq y \leq 1$ about the $y$-axis.
11. Find $\lim _{n \rightarrow \infty} \frac{\sqrt{n}}{\ln n}$
12. Determine whether the series is convergent or divergent.
(a) $\sum_{n=1}^{\infty} \frac{n^{2}}{n^{5 / 7}+1}$
(b) $\sum_{n=1}^{\infty}(-1)^{n} \frac{1}{\sqrt[4]{n}}$
(c) $\sum_{n=1}^{\infty} \frac{n^{2}}{3^{n}}$
(d) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{2}}$
13. Find the radius of convergence an the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{2^{n}(x-3)^{n}}{\sqrt{n+3}}$
14. Find the Taylor series of $f(x)=e^{x}$ at $a=3$.
15. Find the Maclaurin series for $f(x)=x \sin (x / 2)$.
16. Find an equation of the sphere that has center $(1,2,3)$ an passes through the point $(-1,1,-$ $2)$.
17. Given vectors $\vec{a}=<-2,3,4>$ and $\vec{b}=<1,0,3>$. Find
(a) the angle between $\vec{a}$ and $\vec{b}$
(b) the scalar and the vector projections of $\vec{b}$ onto $\vec{a}$
(c) $\vec{a} \times \vec{b}$.
18. Find the volume of the parallelepiped determined by vectors $\vec{a}=<1,0,6>, \vec{b}=<$ $2,3,-8>$, and $\vec{c}=<8,-5,6>$.
19. Represent the point with Cartesian coordinates $(2 \sqrt{3},-2)$ in terms of polar coordinates.
20. Sketch the curve $r=\sin 5 \theta$.
