MATH 152, 501–506, Spring 2011, Sample problems for the Final

- 1. Find the area of the region bounded by $y = x^2$ and $y = \sqrt{x}$.
- 2. Find the volume of the solid obtained by rotating the region bounded by $y = x^2 1$, y = 0, x = 1, x = 2 about the x-axis.
- 3. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, y = 0. x = 1, x = 2 about
 - (a) the y-axis
 - (b) x = 4
- 4. The base of solid S is the triangular region with vertices (0,0), (2,0), and (0,1). Cross-sections perpendicular to the x-axis are semicircles. Find the volume of S.
- 5. A spring has a natural length of 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm, how much work is required to stretch it from 20 cm to 25 cm?
- 6. Find the average value of $f = \sin^2 x \cos x$ on $[-\pi/2, \pi/4]$.
- 7. Evaluate the integral

(a)
$$\int_{0}^{1} x^{2} e^{-x} dx$$

(b)
$$\int \sin^{2} x \cos^{2} x dx$$

(c)
$$\int \sin 3x \cos x dx$$

(d)
$$\int \tan x \sec^{3} x dx$$

(e)
$$\int \frac{x^{2}}{\sqrt{5 - x^{2}}} dx$$

(f)
$$\int \frac{dx}{x^{2}(x^{2} + 1)}$$

(g)
$$\int_{0}^{\infty} \frac{dx}{(x + 2)(x + 3)}$$

(h)
$$\int_{2}^{6} \frac{dx}{\sqrt{x - 2}}$$

- 8. Find the length of the curve $x(t) = 3t t^3$, $y(t) = 3t^2$, $0 \le t \le 2$.
- 9. Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.
- 10. Find the area of the surface obtained by rotating the curve $x = \sqrt{2y y^2}$, $0 \le y \le 1$ about the *y*-axis.

11. Find
$$\lim_{n \to \infty} \frac{\sqrt{n}}{\ln n}$$

12. Determine whether the series is convergent or divergent.

(a)
$$\sum_{n=1}^{\infty} \frac{n^2}{n^{5/7} + 1}$$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[4]{n}}$
(c) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$
(d) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

13. Find the radius of convergence and the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$

- 14. Find the Taylor series of $f(x) = e^x$ at a = 3.
- 15. Find the Maclaurin series for $f(x) = x \sin(x/2)$.
- 16. Find an equation of the sphere that has center (1,2,3) an passes through the point (-1,1,-2).
- 17. Given vectors $\vec{a} = <-2, 3, 4 > \text{and } \vec{b} = <1, 0, 3 >$. Find
 - (a) the angle between \vec{a} and \vec{b}
 - (b) the scalar and the vector projections of \vec{b} onto \vec{a}
 - (c) $\vec{a} \times \vec{b}$.
- 18. Find the volume of the parallelepiped determined by vectors $\vec{a} = < 1, 0, 6 >$, $\vec{b} = < 2, 3, -8 >$, and $\vec{c} = < 8, -5, 6 >$.
- 19. Represent the point with Cartesian coordinates $(2\sqrt{3}, -2)$ in terms of polar coordinates.
- 20. Sketch the curve $r = \sin 5\theta$.