MATH 152, 501-506, Spring 2011, Sample problems for Test 2

1. Evaluate the integral

   (a) $\displaystyle\int \frac{x^2}{\sqrt{5-x^2}}dx$

   (b) $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}dx$

   (c) $\displaystyle\int \frac{dx}{\sqrt{x^2+4x-5}}$

   (d) $\displaystyle\int \frac{dx}{x^2(x^2+1)}$

   (e) $\displaystyle\int_0^{\infty} \frac{dx}{(x+2)(x+3)}$

   (f) $\displaystyle\int_{-\infty}^{1} \frac{dx}{(2x-3)^2}$

   (g) $\displaystyle\int_4^{5} \frac{dx}{(5-x)^{2/5}}$

2. Write out the form of the partial fraction decomposition of the function

$$\frac{x^3+x-1}{(x^2-1)(x+1)(x^2+1)^2}.$$

   Do not determine the numerical values for the coefficients.

3. Find the length of the curve $x(t) = 3t - t^3$, $y(t) = 3t^2$, $0 \le t \le 2$.

4. Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.

5. Find the area of the surface obtained by rotating the curve $x = \sqrt{2y - y^2}$, $0 \le y \le 1$ about the y-axis.

6. Find $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{\ln n}$

7. Find the sum of the series

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

   (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2-4}$

8. Determine whether the series is convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^{5/7}+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

9. Approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ by using the sum of first 4 terms. Estimate the error involved in this approximation.