Section 10.5 **Power series**

A power series is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + ... + c_n x^n + ...$$

Constants c_n are called the **coefficients** of the series. For each fixed x, the series $\sum_{n=0}^{\infty} c_n x^n$ is a series of constants that we can test for convergence or divergence. A power series may converge for some values of x and diverge for other values of x. The sum of the series is a function

$$f(x) = c_0 + c_1 x + c_2 x^2 + ... + c_n x^n + ...$$

whose domain is the set of all x for which the series converges.

More generally, a series of the form $\sum_{n=0}^{\infty} c_n (x-a)^n$ is called a **power series centered at** a or a **power series about** a.

A power series is convergent if $|x-a| < R$, where

$$R = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right|$$

or

$$R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|c_n|}}$$

R is called the **radius of convergence**.

If $R = 0$, then the series converges only at one point $x = a$.

If $R = \infty$, then the series converges for all x.

If $R \neq 0$ and $R < \infty$, then the series converges if $a - R < x < a + R$. Also we need to test the series for convergence at $x = a - R$ and $x = a + R$.

The **interval of convergence** of a power series is the interval that consists of all values of x for which the series is convergent.

Example. Find the radius of convergence and interval of convergence for each of the following series

1. $\sum_{n=0}^{\infty} x^n$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n+2}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 x^n}{10^n}$

5. $\sum_{n=1}^{\infty} \frac{(x-4)^n}{n5^n}$