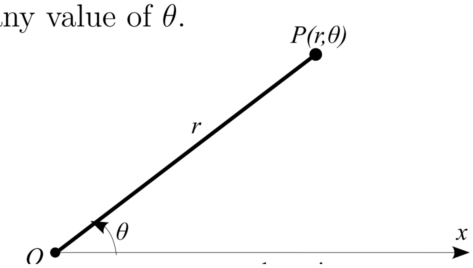
Chapter 13. **Multiple integrals.**
Section 13.4 **Polar coordinates.**

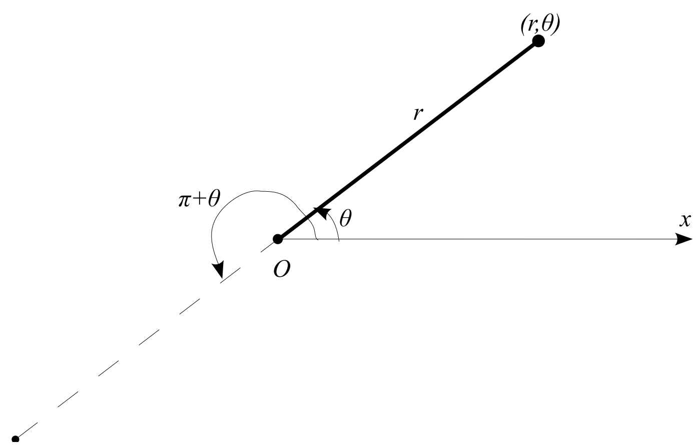
We choose a point in the plane that is called the **pole** (or origin) and labeled O. Then we draw a ray (half-line) starting at O called the **polar axis**. This axis is usually drown horizontally to the right and corresponds to the positive x-axis in Cartesian coordinates.

If P is any point in the plane, let r be the distance from O to P and let θ be the angle (in radians) between the polar axis and the line OP. Then the point P is represented by the ordered pair (r, θ) and r, θ are called **polar coordinates** of P.

We use the convention that an angle is positive if measured in the counterclockwise direction from the polar axis and negative in the clockwise direction. If $P = 0$, then $r = 0$ and we agree that $(0, \theta)$ represents the pole for any value of θ.



We extend the meaning of polar coordinates (r, θ) to the case in which r is negative by agreeing that the points $(-r, \theta)$ and (r, θ) lie on the same line through O and at the same distance $|r|$ from O, but on opposite sides of O.



Example 1. Plot the points whose polar coordinates are given:

(a) $(2, -\pi/7)$ (b) $(-1, \pi/5)$

In the Cartesian coordinate system every point has only one representation, but in the polar coordinate system each point has many representations. Since a complete counterclockwise rotation is given by an angle 2π, the point represented by polar coordinates (r, θ) is also represented by

$$(r, \theta + 2\pi n) \quad \text{and} \quad (-r, \theta + (2n+1)\pi),$$

where n is any integer.

The connection between polar and Cartesian coordinates is

$$x = r\cos\theta \quad y = r\sin\theta$$

and

$$r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}.$$

Equation for θ do not uniquely determine it when x and y are given. Therefore, in converting from Cartesian to polar coordinates, it is not good enough just to find r and θ that satisfy equations. We must choose θ so that the point (r, θ) lies in correct quadrant.

Example 2. Convert the point $(2, 2\pi/3)$ from polar to Cartesian coordinates.

Example 3. Represent the point with Cartesian coordinates $(-1, -\sqrt{3})$ in terms of polar coordinates.

The **graph of a polar equation** $r = f(\theta)$, or more generally, $F(r, \theta) = 0$, consists of all points P that have at least one polar representation (r, θ) whose coordinates satisfy the equation.
Note that:

1. If a polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.

2. If the equation is unchanged when r is replaced by $-r$, the curve is symmetric about the pole.

3. If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \pi/2$.

Example 4. Sketch the curve of each polar equation

(a) $r = 1 - \cos\theta$

(b) $r = \sin 2\theta$.