Section 7.5 **Average value of a function**

Let us try to compute the average value of a function $y = f(x)$, $a \le x \le b$. We start by dividing the interval $[a, b]$ into n equal subintervals, each with length $\Delta x = (b-a)/n$ and choose points x_i^* in successive subintervals. Then the average of the numbers $f(x_1^*)$, $f(x_2^*)$,...,$f(x_n^*)$, is

$$\frac{f(x_1^*) + f(x_2^*) + ... + f(x_n^*)}{n}$$

Since $n = (b-a)\Delta x$,

$$\frac{f(x_1^*) + f(x_2^*) + ... + f(x_n^*)}{\frac{b-a}{\Delta x}} = \frac{1}{b-a}(f(x_1^*)\Delta x + f(x_2^*)\Delta x + ... + f(x_n^*)\Delta x)$$

The limiting value as $n \to \infty$ is

$$\lim_{n\to\infty} \frac{1}{b-a} \sum_{i=1}^{n} f(x_i^*)\Delta x = \frac{1}{b-a} \int_a^b f(x)dx$$

We define the **average value of** f on the interval $[a, b]$ as

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x)dx$$

Example 1. Find the average value of $f(x) = \sin^2 x \cos x$ on $[\pi/2, \pi/4]$.

Example 2. Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

Mean value theorem for integrals If f continuous on $[a, b]$, then there exist a number c in $[a, b]$ such that

$$\int_a^b f(x)dx = f(c)(b - a)$$

The geometric interpretation of this theorem for *positive* functions $f(x)$, there is a number c such that the rectangle with base $[a, b]$ and height $f(c)$ has the same area as a region under the graph of f from a to b.

Example 3. Find the average value of the function $f(x) = 4 - x^2$ on the interval $[0, 2]$. Find c $(0 \leq c \leq 2)$ such that $f_{ave} = f(c)$.