

Chapter 8. Techniques of integration
Section 8.1 Integration by parts

The formula for integration by parts for indefinite integrals is

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

The formula for integration by parts for definite integrals is

$$\int_a^b f(x)g'(x)dx = f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$$

Example 1. Find the integral.

1. $\int x \cos 3x \, dx$

2. $\int \ln x \, dx$

3. $\int_0^1 t^2 e^t \, dt$

4. $\int e^x \cos x \, dx$

Example 2.

1. Prove the following reduction formulas:

(a) $\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$

(b) $\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$

$$(c) \int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$$

2. Use the reduction formulas 1.1–1.3 to evaluate

(a) $\int (\ln x)^3 dx$

(b) $\int (x^2 - 2x)e^x dx$

Example 3. Use the methods of cylindrical shells to find the volume generated by rotating the region bounded by $y = e^{-x}$, $y = 0$, $x = -1$, $x = 0$ about $x = 1$.