MATH 152, 525-530, 534-536, Spring 2013, Sample problems for Test 3

1. Which of the following series is convergent?

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^{5/7}+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

2. Approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ by using the sum of first 4 terms. Estimate the error involved in this approximation.

3. Approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} ne^{-n^2}$ by using the sum of first 4 terms. Estimate the error involved in this approximation.

4. Which of the following series is absolutely convergent?

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n}{\sqrt{n-2}}$

   (d) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n}\frac{2^{2n}}{3^{3n}}$

5. Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$.

6. Find the power series representation for the function $f(x) = \ln(1-2x)$ centered at 0.

7. Find the Taylor series for $f(x) = xe^{2x}$ at $x = 2$.

8. Find the Maclaurin series for $f(x) = x\sin(x^3)$.

9. Find the sum of the series

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^2}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

10. Evaluate the indefinite integral as a power series $\int e^{x^2}\,dx$.

11. Approximate $f(x) = \sin x$ by a Taylor polynomial of degree 3 at $\pi/4$. How accurate is this approximation if $0 \leq x \leq \pi/2$?

12. Find radius and center of sphere given by the equation $x^2 + y^2 + z^2 = 6x + 4y + 10z$

13. Find the angle between the vectors $\vec{a} = \vec{\imath} + \vec{\jmath} + 2\vec{k}$ and $\vec{b} = 2\vec{\jmath} - 3\vec{k}$.

14. Find the directional cosines for the vector $\vec{a} = -2\vec{\imath} + 3\vec{\jmath} + \vec{k}$.

15. Find the scalar and the vector projections of the vector $< 2, -3, 1 >$ onto the vector $< 1, 6, -2 >$.