MATH 152, 525-530, 534-536, Spring 2013, Answers for sample problems for Test 3.

1. (a) diverges
   (b) converges
   (c) converges

2. $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2} \approx e^{-1} + 2e^{-4} + 3e^{-9} + 4e^{-16}.\ \frac{e^{-25}}{2} \le R_4 \le \frac{e^{-16}}{2}.$

3. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} ne^{-n^2} \approx e^{-1} - 2e^{-4} + 3e^{-9} - 4e^{-16}.\ |R_4| \le 5e^{-25}.$

4. (a) absolutely convergent
   (b) convergent but not absolutely convergent
   (c) divergent
   (d) absolutely convergent

5. $R = \dfrac{1}{2},\ \left[-\dfrac{5}{2}, \dfrac{7}{2}\right)$

6. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^{n+1}}{n+1}$

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(n2^{n-1} + 2^{n+1})e^4}{n!}(x-2)^n$

8. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+4}}{(2n+1)!}$

9. (a) $x^2 e^{-1}$
   (b) $\dfrac{\sqrt{3}}{2}$

10. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!(2n+1)} + C$

11. $\displaystyle\sin x \approx \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3.\ |R_3| \le \frac{\pi^4}{6144}.$

12. $R = \sqrt{38},\ C(3, 2, 5).$

13. $\displaystyle\theta = \cos^{-1}\left(-\frac{4}{\sqrt{78}}\right)$

14. $\displaystyle\cos\alpha = -\frac{2}{\sqrt{14}}, \cos\beta = \frac{3}{\sqrt{14}}, \cos\gamma = \frac{1}{\sqrt{14}}$

15. $\displaystyle-\frac{18}{\sqrt{41}},\ \left\langle -\frac{18}{41}, -\frac{108}{41}, \frac{36}{41}\right\rangle$