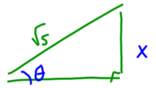
Some Review problem

1) $\int \frac{x^2}{\sqrt{5-x^2}}\, dx$ $\qquad 5-x^2$



$\sqrt{5-x^2} = \sqrt{5}\cos\theta$

$\boxed{\sin\theta = \frac{x}{\sqrt{5}}}$

$\begin{cases} x = \sqrt{5}\sin\theta \\ dx = \sqrt{5}\cos\theta\, d\theta \end{cases}$

$$= \int \frac{(\sqrt{5}\sin\theta)^2}{\cancel{\sqrt{5}\cos\theta}} (\cancel{\sqrt{5}\cos\theta}\, d\theta)$$

$$= \int 5\sin^2\theta\, d\theta$$

power is even

$\begin{cases} \sin^2\theta = \frac{1}{2}[1-\cos 2\theta] \\ \cos^2\theta = \frac{1}{2}[1+\cos 2\theta] \end{cases}$

$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$$= \frac{5}{2}\int 1-\cos 2\theta\, d\theta = \frac{5}{2}\Big(\theta - \underbrace{\tfrac{1}{2}\sin 2\theta}_{\sin\theta\cos\theta}\Big) + C$$

$$= \frac{5}{2}\left(\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) - \left(\frac{x}{\sqrt{5}}\right)\left(\frac{\sqrt{5-x^2}}{\sqrt{5}}\right)\right) + C$$

$$= \frac{5}{2}\left(\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) - \frac{x\sqrt{5-x^2}}{5}\right) + C$$

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2) $\left[\int_0^\infty \frac{dx}{(x+2)(x+3)}\right.$

$x=-2,\ x=-3$ are not in $[0,\infty)$

$$= \lim_{a\to\infty}\int_0^a \frac{dx}{(x+2)(x+3)}$$

Need Partial fractions for

$\to \frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$

$1 = A(x+3) + B(x+2)$

$x=-3 \Rightarrow 1 = B(-1) \Rightarrow B=-1$

$x=-2 \Rightarrow 1 = A(1) \Rightarrow A=1$

$\boxed{\frac{1}{(x+2)(x+3)} = \frac{1}{x+2} - \frac{1}{x+3}}$

$$= \lim_{a\to\infty}\int_0^a \left(\frac{1}{x+2} - \frac{1}{x+3}\right)dx$$

$$= \lim_{a\to\infty}\Big[\ln|x+2| - \ln|x+3|\Big]_0^a$$

$$= \lim_{a\to\infty}\Big[\overset{\infty}{\ln|a+2|} - \overset{-\infty}{\ln|a+3|} - \ln|0+2| + \ln|0+3|\Big]$$

$$= \lim_{a\to\infty}\left[\ln\left|\frac{a+2}{a+3}\right| - \ln 2 + \ln 3\right]$$

goes to 1 (by l'Hopital)

$$= \cancel{\ln 1}^{\,0} - \ln 2 + \ln 3$$

$$= \ln \frac{3}{2}$$

The improper integral converges to $\ln \frac{3}{2}$.

3) length of the curve $x = 3t - t^3$, $y = 3t^2$ $0 \le t \le 2$.

$x' = 3 - 3t^2$ $y' = 6t$ a b

$$L = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

$$= \int_0^2 \sqrt{(3-3t^2)^2 + (6t)^2}\, dt = \int_0^2 \sqrt{3^2 \underbrace{- 2(3)(3t^2)}_{-18t^2} + \underbrace{(3t^2)^2}_{9t^4} + 36t^2}\, dt$$

$$= \int_0^2 \sqrt{3^2 + 2(3)(3t^2) + (3t^2)^2}\, dt$$

$$= \int_0^2 \sqrt{(\underbrace{3+3t^2}_{>0})^2}\, dt = \int_0^2 |\overbrace{3+3t^2}^{>0}|\, dt = \int_0^2 3+3t^2\, dt$$

$$= 3t + t^3 \Big|_0^2 = 3(2) + 8 - 0 = 14$$

4) Are the following series absolutely convergent?

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$$

Ratio test

$$L = \lim_{n\to\infty} \left| \frac{(-3)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-3)^n} \right|$$

$n+1$

$$= \lim_{n\to\infty} \frac{|-3|}{n+1} = 0 < 1$$

The series is absolutely convergent

$\Downarrow$

Convergent

side note

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad (R = \infty)$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{(-3)^n}{n!} = e^{-3} = \frac{1}{e^3}$$

$$\sum_{n=3}^{\infty} (-1)^{n-1} \frac{n}{\sqrt{n-2}}$$

$$\sum_{n=3}^{\infty} \left| (-1)^{n-1} \frac{n}{\sqrt{n-2}} \right| = \sum_{n=3}^{\infty} \frac{n}{\sqrt{n-2}}$$

$a_n \longrightarrow \infty \ne 0$

the series diverges by Test for Divergence

5) Find $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!}$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$x = \frac{\pi}{6}$

$$= \cos \tfrac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Title : May 4-10:51 AM (Page 2 of 2)