1) $\int_0^1 x^2 e^{-x}\,dx$

$\begin{cases} u = x^2 \\ du = 2x\,dx \end{cases}$ $\begin{cases} dv = e^{-x}\,dx \\ v = -e^{-x} \end{cases}$

$\int u\,dv = uv - \int v\,du$

$x^2(-e^{-x})\Big|_0^1 - \int_0^1 (-e^{-x})(2x\,dx)$

$= (-e^{-1} - 0) + \int_0^1 2x e^{-x}\,dx$

$\begin{cases} u = 2x \\ du = 2\,dx \end{cases}$ $\begin{cases} dv = e^{-x}\,dx \\ v = -e^{-x} \end{cases}$

$= -\frac{1}{e} + \left(2x(-e^{-x})\right)\Big|_0^1 - \int_0^1 -e^{-x}(2\,dx)$

$= -\frac{1}{e} - \frac{2}{e} + 2\int_0^1 e^{-x}\,dx = -\frac{3}{e} + \left[-2e^{-x}\right]_0^1 = -\frac{3}{e} + \left[-\frac{2}{e} + 2\right]$

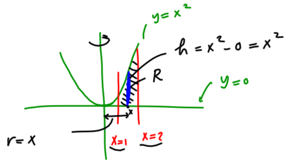
$= 2 - \frac{5}{e} = \frac{2e-5}{e} > 0$

2) set up integrals Volume of the solid of rotation obtained by rotating the region bdd by $y = x^2$, $y = 0$, $x = 1$, $x = 2$ about

a) the y-axis b) $x = 4$ c) $y = -2$

a) about y-axis:

shell method: $V = 2\pi \int_a^b r\,h\,dx$



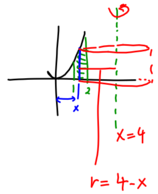
$= 2\pi \int_1^2 x(x^2)\,dx = 2\pi \int_1^2 x^3\,dx = 2\pi \frac{x^4}{4}\Big|_1^2 = 2\pi\left(\frac{16}{4} - \frac{1}{4}\right)$

$= 2\pi\left(\frac{15}{4}\right) = \frac{15\pi}{2}$

b) about $x = 4$

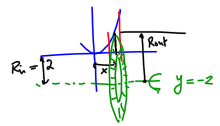
$V = 2\pi \int_1^2 (4-x)\,x^2\,dx$

$= 2\pi \int_1^2 4x^2 - x^3\,dx = \cdots$



c) Use washers

$V = \pi \int_1^2 (R_{out}^2 - R_{in}^2)\,dx$



$= \pi \int_1^2 (x^2+2)^2 - (2)^2\,dx = \pi \int_1^2 x^4 + 4x^2 + 4 - 4\,dx$

$= \pi \int_1^2 x^4 + 4x^2\,dx = \cdots$

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3) Evaluate $\int_4^5 \frac{dx}{(5-x)^{2/5}}$ $x=5$ improper

$$= \lim_{a\to 5-} \int_4^a \underbrace{(5-x)}_{\text{linear}}^{-2/5}\,dx = \lim_{a\to 5-} \frac{1}{(-1)} \frac{(5-x)^{-2/5+1}}{-2/5+1}\Bigg|_4^a$$

$-2/5+1 = 3/5$

derivative of the linear term (only when linear!!)

a continuous function of a b/c $3/5>0$

$$= \lim_{a\to 5-} \left[-\tfrac{5}{3}(5-x)^{3/5}\right]_4^a = \lim_{a\to 5-}\left[-\tfrac{5}{3}(\underbrace{5-a}_{\text{goes to } 0})^{3/5 > 0} + \tfrac{5}{3}(\underbrace{5-4}_{1})^{3/5}\right]$$

$$= -\tfrac{5}{3}(0) + \tfrac{5}{3} = \tfrac{5}{3}$$

4) Find the sum

$$\sum_{n=2}^{\infty} \frac{(-1)^n x^2}{n!} = x^2 \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} = x^2\left[e^{-1} - (1 + (-1))\right] = \frac{x^2}{e}$$

fixed

not a power series !!

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots$$

varies

a power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n}}{n!} = \sum_{n=2}^{\infty} \frac{(-x^2)^n}{n!} = e^{-x^2} - [1 - x^2]$$

$$= e^{-x^2} - 1 + x^2$$

5) Convergence:

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$$

positive use comparison

$$0 \le \frac{\cos^2 n}{3^n} \le \frac{1}{3^n}$$

$\sum_{n=1}^{\infty} \frac{1}{3^n}$ converges ($-1 < r = 1/3 < 1$)

$\Rightarrow \sum \frac{\cos^2 n}{3^n}$ converges by the Direct Comparison Test.