MATH 152, 513–515, 519–524, Sample problems for Exam 2, Spring 2015

1. Evaluate the integral

   (a) $\displaystyle\int \frac{x^2}{\sqrt{5-x^2}}dx$

   (b) $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}dx$

   (c) $\displaystyle\int \frac{dx}{\sqrt{x^2+4x-5}}$

   (d) $\displaystyle\int \frac{dx}{x^2(x^2+1)}$

   (e) $\displaystyle\int \frac{x^2+3x-1}{x-1}dx$

   (f) $\displaystyle\int_0^{\infty} \frac{dx}{(x+2)(x+3)}$

   (g) $\displaystyle\int_{-\infty}^{1} \frac{dx}{(2x-3)^2}$

   (h) $\displaystyle\int_4^{5} \frac{dx}{(5-x)^{2/5}}$

2. Write out the form of the partial fraction decomposition of the function

$$\frac{x^3+x-1}{(x^2-1)(x+1)(x^2+1)^2}.$$

   Do not determine the numerical values for the coefficients.

3. Use the Comparison Theorem to determine which of the following integrals is convergent.

   (a) $\displaystyle\int_3^{\infty} \frac{3+\sin x}{x}dx$

   (b) $\displaystyle\int_1^{\infty} \frac{2+\cos x}{x^2}dx$

4. Find the length of the curve $x(t) = 3t - t^3$, $y(t) = 3t^2$, $0 \le t \le 2$.

5. Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.

6. Find the area of the surface obtained by rotating the curve $x = \sqrt{2y-y^2}$, $0 \le y \le 1$ about the y-axis.

7. Which sequence is both bounded and increasing?

   (a) $a_n = 1 - \dfrac{2}{n}$

   (b) $a_n = \ln n$

   (c) $a_n = \sin(2\pi n)$

   (d) $a_n = e^{-n}$

8. Find the following limits

   (a) $\lim\limits_{n\to\infty} \dfrac{\sqrt{n}}{\ln n}$

   (b) $\lim\limits_{n\to\infty} \dfrac{1-2n^2}{\sqrt[3]{n^6+1}+2n^2}$

   (c) $\lim\limits_{n\to\infty} (\sqrt{n+1}-\sqrt{n})$

9. Find the sum of the series

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

   (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2-4}$