3. Find the length of the curve $x(t) = 3t - t^3$, $y(t) = 3t^2$, $0 \leq t \leq 2$.

$$L = \int_0^2 \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$x'(t) = 3\cancel{t} - 3t^2$$

$$y'(t) = 6t$$

$$= \int_0^2 \sqrt{(3-3t^2)^2 + 36t^2}\, dt$$

$$= \int_0^2 \sqrt{9 - 18t^2 + 9t^4 + 36t^2}\, dt$$

$$= \int_0^2 \sqrt{9 + 18t^2 + 9t^4}\, dt$$

$$= \int_0^2 \sqrt{(3+3t^2)^2}\, dt$$

$$= \int_0^2 (3+3t^2)\, dt$$

$$= \left(3t + \frac{3t^3}{3}\right)\Big|_0^2$$

$$= 6 + 8$$

$$= \boxed{14}$$

4. Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.

$$S_x = 2\pi \int_0^2 y(x)\sqrt{1+[y'(x)]^2}\,dx$$

$$y'(x) = 3x^2$$

$$= 2\pi \int_0^2 x^3 \sqrt{1+9x^4}\,dx$$

$u = 1+9x^4$
$du = 36x^3\,dx$
$x=0 \longrightarrow u=1$
$x=2 \longrightarrow u = 1+9(16)$
$= 145$

$$= \frac{2\pi}{36}\int_1^{145} \sqrt{u}\,du$$

$$= \frac{\pi}{18}\,\frac{2}{3}u^{3/2}\Big|_1^{145}$$

$$= \boxed{\frac{\pi}{27}\left((145)^{3/2}-1\right)}$$

5. Find the area of the surface obtained by rotating the curve $x = \sqrt{2y - y^2}$, $0 \le y \le 1$ about the y-axis.

$$S_y = 2\pi \int_0^1 x(y)\sqrt{1 + (x'(y))^2}\,dy$$

$$x'(y) = \frac{1}{2\sqrt{2y-y^2}}(2-2y) = \frac{1-y}{\sqrt{2y-y^2}}$$

$$= 2\pi \int_0^1 \sqrt{2y-y^2}\sqrt{1 + \frac{(1-y)^2}{2y-y^2}}\,dy$$

$$= 2\pi \int_0^1 \sqrt{2y-y^2}\sqrt{\frac{2y-y^2+1-2y+y^2}{2y-y^2}}\,dy$$

$$= 2\pi \int_0^1 \sqrt{2y-y^2}\sqrt{\frac{1}{2y-y^2}}\,dy$$

$$= \boxed{2\pi}$$

6. Find the following limits

(a) $\lim_{n\to\infty} \frac{\sqrt{n}}{\ln n} = \frac{\infty}{\infty} \overset{L'H.R.}{=\!=} \lim_{n\to\infty} \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n}}$

$= \lim_{n\to\infty} \frac{n}{2\sqrt{n}}$

$= \lim_{n\to\infty} \frac{\sqrt{n}}{2}$

$= \infty$

(b) $\lim_{n\to\infty} \frac{1-2n^2}{\sqrt[3]{n^6+1}+2n^2}$

$$= \lim_{n\to\infty} \frac{n^2\left(\frac{1}{n^2}-2\right)}{\sqrt[3]{n^6\left(1+\frac{1}{n^6}\right)}+2n^2}$$

$$= \lim_{n\to\infty} \frac{-2n^2}{\sqrt[3]{n^6}+2n^2}$$

$$= \lim_{n\to\infty} \frac{-2n^2}{n^2+2n^2}$$

$$= \lim_{n\to\infty} \frac{-2n^2}{3n^2}$$

$$= \boxed{-\frac{2}{3}}$$

(c) $\lim_{n\to\infty}(\sqrt{n+1}-\sqrt{n})$

$$= \lim_{n\to\infty} \frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$= \boxed{0}$$

7. Find the sum of the series

(a) $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

$$= \sum_{n=1}^{\infty} \frac{2 \cdot 2^{2n}}{\frac{1}{3} 3^{3n}}$$

$$= \sum_{n=1}^{\infty} 6 \left(\frac{2^2}{3^3}\right)^n$$

$$= \sum_{n=1}^{\infty} 6 \left(\frac{4}{27}\right)^n$$

$$= \sum_{n=1}^{\infty} 6 \left(\frac{4}{27}\right) \left(\frac{4}{27}\right)^{n-1}$$

$$= \frac{6 \frac{4}{27}}{1 - \frac{4}{27}}$$

$$= \frac{\frac{24}{27}}{\frac{23}{27}}$$

$$= \boxed{\frac{24}{23}}$$

(b) $\sum_{n=3}^{\infty} \frac{1}{n^2-4}$ Partial fractions:

$$\frac{1}{n^2-4} = \frac{1}{(n-2)(n+2)} = \frac{A}{n-2} + \frac{B}{n+2}$$

$$= \frac{A(n+2)+B(n-2)}{(n-2)(n+2)}$$

$$1 = A(n+2) + B(n-2)$$

$n=-2$: $1 = -4B \longrightarrow B = -\frac{1}{4}$

$n=2$: $1 = 4A \longrightarrow A = \frac{1}{4}$

$$\frac{1}{n^2-4} = \frac{1}{4}\left(\frac{1}{n-2} - \frac{1}{n+2}\right)$$

Partial sums:

$$S_3 = \frac{1}{4}\left(\frac{1}{1} - \frac{1}{5}\right) = a_3$$

$$a_3 + a_4 = S_4 = \frac{1}{4}\left(\frac{1}{1} - \frac{1}{5}\right) + \frac{1}{4}\left(\frac{1}{2} - \frac{1}{6}\right)$$

$$= \frac{1}{4}\left(\frac{1}{1} + \frac{1}{2} - \frac{1}{5} - \frac{1}{6}\right)$$

$$S_5 = a_3 + a_4 + a_5 = \frac{1}{4}\left(\frac{1}{1} + \frac{1}{2} - \frac{1}{5} - \frac{1}{6}\right) + \frac{1}{4}\left(\frac{1}{3} - \frac{1}{7}\right)$$

$$S_6 = a_3 + a_4 + a_5 + a_6 = \frac{1}{4}\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} - \frac{1}{5} - \frac{1}{6} - \frac{1}{7}\right)$$
$$+ \frac{1}{4}\left(\frac{1}{4} - \frac{1}{8}\right)$$

$$S_7 = a_3 + a_4 + a_5 + a_6 + a_7 = \frac{1}{4}\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \cancel{\frac{1}{5}} - \frac{1}{6} - \frac{1}{7} - \frac{1}{8}\right)$$
$$+ \frac{1}{4}\left(\cancel{\frac{1}{5}} - \frac{1}{9}\right)$$

$$= \frac{1}{4}\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{6} - \frac{1}{7} - \frac{1}{8} - \frac{1}{9}\right)$$

$$S_n = \frac{1}{4}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{n-1}-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$S = \lim_{n\to\infty} S_n = \frac{1}{4}\lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{n-1}-\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$= \frac{1}{4}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)$$

$$= \frac{1}{4}\,\frac{24+12+8+6}{24}$$

$$= \frac{1}{4}\,\frac{50}{24}$$

$$= \boxed{\frac{25}{48}}$$