

## Section 10.6. Representations of functions as a power series

Geometric series.

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad |x| < 1.$$

**Example 1.** Find a power series representation for the function and determine the interval of convergence.

1.  $\frac{1}{1+x}$

2.  $\frac{1}{1-x^2}$

3.  $\frac{1}{1+4x^2}$

$$4. \frac{1+x^2}{1-x^2}$$

**Term-by-term differentiation and integration.** If the power series  $\sum_{n=0}^{\infty} c_n(x-a)^n$  has radius of convergence  $R > 0$ , then the function  $f$  defined by

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots + c_n(x-a)^n + \dots = \sum_{n=0}^{\infty} c_n(x-a)^n$$

is differentiable (and therefore continuous) on the interval  $(a-R, a+R)$  and

$$f'(x) = c_1 + 2c_2(x-a) + \dots + nc_n(x-a)^{n-1} + \dots = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$$

$$\int f(x)dx = C + c_0(x-a) + c_1 \frac{(x-a)^2}{2} + \dots + c_n \frac{(x-a)^{n+1}}{n+1} + \dots =$$

$$C + \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$$

The radii of convergence of these series are  $R$ . This does not mean that the interval of convergence remains the same.

**Example 2.** Find a power series representation for the function and determine the radius of convergence.

$$1. \frac{1}{(1+x)^2}$$

2.  $\ln(1+x)$

**Example 3.** Evaluate an indefinite integral  $\int \tan^{-1}(x^2) dx$  as a power series.

**Example 4.** Use a power series to approximate the integral

$$\int_0^{1/2} \tan^{-1}(x^2) dx$$

to six decimal places.