Table of indefinite integrals

1. $\int adx = ax + C$, a is a constant,
2. $\int xdx = \frac{x^2}{2} + C$,
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$,
4. $\int \frac{1}{x} dx = \ln|x| + C$,
5. $\int \mathrm{e}^x dx = \mathrm{e}^x + C$,
6. $\int \mathrm{a}^x dx = \frac{\mathrm{a}^x}{\ln \mathrm{a}} + C$,
7. $\int \sin xdx = -\cos x + C$,
8. $\int \cos xdx = \sin x + C$,
9. $\int \tan xdx = -\ln|\cos x| + C$,
10. $\int \cot xdx = \ln|\sin x| + C$,
11. $\int \sec^2 xdx = \tan x + C$,
12. $\int \csc^2 xdx = -\cot x + C$,
13. $\int \sec x \tan xdx = \sec x + C$,
14. $\int \csc x \cot x = -\csc x + C$,
15. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$,
16. $\int \frac{1}{1+x^2} dx = \arctan x + C$.

Definition of a definite integral

If f is a function defined on a closed interval $[a,b]$, let P be a partition of $[a,b]$ with partition points x_0, x_1,...,x_n, where

$$a = x_0 < x_1 < x_2 < ... < x_n = b$$

Choose points $x_i^* \in [x_{i-1}, x_i]$ and let $\Delta x_i = x_i - x_{i-1}$ and $\|P\| = \max\{\Delta x_i\}$. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

if this limit exists. If the limit does exist, then f is called **integrable** on the interval $[a,b]$.

In the notation $\int_a^b f(x)dx$, $f(x)$ is called the **integrand** and a and b are called the limits of integration; a is the **lower limit** and b is the **upper limit**.

The procedure of calculating an integral is called **integration**.

Properties of the definite integral

1. $\int_a^b cdx = c(b-a)$, where c is a constant.
2. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, where c is a constant.
3. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$.
4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $a < c < b$.
5. $\int_a^b f(x)dx = -\int_b^a f(x)dx$.
6. If $f(x) \geq 0$ for $a < x < b$, then $\int_a^b f(x)dx \geq 0$.
7. If $f(x) \geq g(x)$ for $a < x < b$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.

8. If $m \le f(x) \le M$ for $a < x < b$, then $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$.

9. $\left| \int_a^b f(x)dx \right| \le \int_a^b |f(x)|dx$

Section 6.4 The fundamental theorem of calculus.

Suppose f is continuous on $[a,b]$.

1. If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$.

2. $\boxed{\int_a^b f(x)dx = F(b) - F(a) = F(x)|_a^b,}$ where F is an antiderivative of f.

Example 1. Evaluate the integral.

1. $\int_2^6 \frac{1+\sqrt{y}}{y^2} dy = \int_2^6 (1+y^{1/2})y^{-2}\,dy = \int_2^6 \left(y^{-2} + y^{\overbrace{1/2-2}^{-3/2}}\right) dy$

$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1}$

$= \int_2^6 (y^{-2} + y^{-3/2}) dy = \left[\frac{y^{-2+1}}{-2+1} + \frac{y^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \right]_2^6$

$= \left[-y^{-1} + \frac{y^{-1/2}}{-1/2} \right]_2^6 = \left[-\frac{1}{y} - \frac{2}{y^{1/2}} \right]_2^6$

$= \boxed{-\frac{1}{6} - \frac{2}{6^{1/2}} - \left(-\frac{1}{2} - \frac{2}{2^{1/2}} \right)}$

2. $\int_0^2 f(x)dx$, where $f(x) = \begin{cases} x^4 & 0 \le x < 1 \\ x^5 & 1 \le x \le 2 \end{cases}$

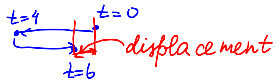
$\int_0^2 f(x)dx = \int_0^1 \overbrace{f(x)}^{x^4}dx + \int_1^2 \overbrace{f(x)}^{x^5}dx = \int_0^1 x^4 dx + \int_1^2 x^5 dx$

$= \frac{x^{4+1}}{4+1}\Big]_0^1 + \frac{x^{5+1}}{5+1}\Big]_1^2$

$= \frac{x^5}{5}\Big]_0^1 + \frac{x^6}{6}\Big]_1^2 = \frac{1}{5} + \frac{2^6}{6} - \frac{1}{6} = \boxed{\frac{1}{5} + \frac{64}{6} - \frac{1}{6}}$

Example 2. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - 2t - 8$.

1. Find the displacement of the particle during the time period $1 \le t \le 6$.



$$\text{displacement} = \left| \int_1^6 v(t)\,dt \right| = \left| \int_1^6 (t^2 - 2t - 8)\,dt \right|$$

$$= \left| \left[\frac{t^3}{3} - \frac{2t^2}{2} - 8t \right]_1^6 \right|$$

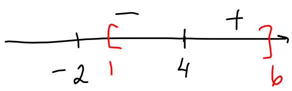
$$= \left| \frac{6^3}{3} - 6^2 - 8(6) - \left(\frac{1}{3} - 1 - 8 \right) \right| = \left| 72 - 36 - 48 - \frac{1}{3} + 9 \right| = \left| -\frac{10}{3} \right|$$

$$= \boxed{\frac{10}{3}}$$

2. Find the distance traveled during this time period.

find t such that $v(t) \ge 0$ / $v(t) \le 0$, $1 \le t \le 6$

$$t^2 - 2t - 8 \ge 0$$
$$(t-4)(t+2) \ge 0$$



$v(0) = -8 < 0$

$v(5) = 7 > 0$

$v(t) \le 0$ on $[1, 4]$

$v(t) \ge 0$ on $[4, 6]$

$$\text{distance} = \int_1^4 (-v(t))\,dt + \int_4^6 v(t)\,dt$$