MATH 152, 549 – 551, Sample problems for Exam 2, Spring 2016

1. Evaluate the integral

   (a) $\displaystyle\int \frac{x^2}{\sqrt{5-x^2}}dx$

   (b) $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}dx$

   (c) $\displaystyle\int \frac{dx}{\sqrt{x^2+4x-5}}$

   (d) $\displaystyle\int \frac{dx}{x^2(x^2+1)}$

   (e) $\displaystyle\int \frac{x^2+3x-1}{x-1}dx$

   (f) $\displaystyle\int_0^\infty \frac{dx}{(x+2)(x+3)}$

   (g) $\displaystyle\int_{-\infty}^1 \frac{dx}{(2x-3)^2}$

   (h) $\displaystyle\int_4^5 \frac{dx}{(5-x)^{2/5}}$

2. Write out the form of the partial fraction decomposition of the function

$$\frac{x^3+x-1}{(x^2-1)(x+1)(x^2+1)^2}.$$

   Do not determine the numerical values for the coefficients.

3. Use the Comparison Theorem to determine which of the following integrals is convergent.

   (a) $\displaystyle\int_3^\infty \frac{3+\sin x}{x}dx$

   (b) $\displaystyle\int_1^\infty \frac{2+\cos x}{x^2}dx$

   (c) $\displaystyle\int_1^\infty \frac{dx}{x+e^{3x}}$

4. Find the length of the curve $x(t)=3t-t^3$, $y(t)=3t^2$, $0\le t\le 2$.

5. Find the area of the surface obtained by rotating the curve $y=x^3$, $0\le x\le 2$ about the x-axis.

6. Find the area of the surface obtained by rotating the curve $x = \sqrt{2y - y^2}$, $0 \le y \le 1$ about the y-axis.

7. Which sequence is both bounded and increasing?

   (a) $a_n = 1 - \dfrac{2}{n}$

   (b) $a_n = \ln n$

   (c) $a_n = \sin(2\pi n)$

   (d) $a_n = e^{-n}$

8. Find the following limits

   (a) $\displaystyle\lim_{n\to\infty} \frac{(-1)^n}{n^3}$

   (b) $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{\ln n}$

   (c) $\displaystyle\lim_{n\to\infty} \frac{1 - 2n^2}{\sqrt[3]{n^6 + 1} + 2n^2}$

   (d) $\displaystyle\lim_{n\to\infty} \left(\frac{1}{3}\ln(n^3 + 5n - 2) - \ln(2 - n)\right)$

9. The sequence defined by $a_1 = 2$ and $a_{n+1} = 5 - \dfrac{4}{a_n}$ is increasing and bounded above. Find its limit.

10. If the series $\displaystyle\sum_{n=1}^{\infty} a_n$ has a partial sum of $s_n = \dfrac{2n+3}{3n-1}$, find a_3 and the sum of the series.

11. Find the sum of the series

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

    (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2 - 4}$

12. Which of the following statements is true for the series $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{\sqrt{1 + 4n^2}}$?

    I. It converges by the Divergence Test.

    II. It converges to $\dfrac{3}{2}$.

    III. It diverges.