# MATH152, 525–530, 534–536 Spring 2013, Sample problems for the Final

1. Find the area of the region bounded by $y = x^2 + 1$, $y = 3 - x^2$, $x = -2$, and $x = 2$.

2. Find the volume of the solid obtained by rotating the region bounded by $y = x^2 - 1$, $y = 0$, $x = 1$, $x = 2$ about the $x$-axis.

3. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 0$. $x = 1$, $x = 2$ about

   (a) the $y$-axis

   (b) $x = 4$

4. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft hight. How much work is done in pulling the rope to the top of the building?

5. A spring has a natural length of 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm, how much work is required to stretch it from 20 cm to 25 cm?

6. Find the average value of $f = \sin^2 x \cos x$ on $[-\pi/2, \pi/4]$.

7. Evaluate the integral

   (a) $\int t^2 \cos(1 - t^3) dt$

   (b) $\int \dfrac{x^2}{\sqrt{1-x}} dx$

   (c) $\int_0^1 x^2 e^{-x} dx$

   (d) $\int \sin^3 x \cos^4 x \; dx$

   (e) $\int_0^{\pi/8} \sin^2(2x) \cos^3(2x) \; dx$

   (f) $\int \sin^2 x \cos^4 x \; dx$

   (g) $\int_0^{\pi/4} \tan^4 x \sec^2 x \; dx$

   (h) $\int \tan x \sec^3 x \; dx$

   (i) $\int \sin 3x \cos x \; dx$

   (j) $\displaystyle\int \frac{x^2}{\sqrt{5 - x^2}} dx$

   (k) $\displaystyle\int \frac{x^3}{\sqrt{x^2 + 4}} dx$

   (l) $\displaystyle\int \frac{dx}{\sqrt{x^2 + 4x - 5}}$

   (m) $\displaystyle\int \frac{dx}{x^2(x^2 + 1)}$

(n) $\displaystyle\int \frac{x^2+3x-1}{x-1}dx$

(o) $\displaystyle\int_0^{\infty} \frac{dx}{(x+2)(x+3)}$

(p) $\displaystyle\int_{-\infty}^{1} \frac{dx}{(2x-3)^2}$

(q) $\displaystyle\int_4^{5} \frac{dx}{(5-x)^{2/5}}$

8. Find the length of the curve $x(t) = 3t - t^3$, $y(t) = 3t^2$, $0 \le t \le 2$.

9. Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the $x$-axis.

10. Find the area of the surface obtained by rotating the curve $x = \sqrt{2y - y^2}$, $0 \le y \le 1$ about the $y$-axis.

11. Find the following limits

    (a) $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{\ln n}$

    (b) $\displaystyle\lim_{n\to\infty} \frac{1-2n^2}{\sqrt[3]{n^6+1}+2n^2}$

    (c) $\displaystyle\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n})$

12. Find the sum of the series

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

    (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^2}{n!}$

    (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

13. Which of the following series is convergent?

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^{5/7}+1}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$

    (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

14. Which of the following series is absolutely convergent?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{\sqrt{n-2}}$

(d) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n} \frac{2^{2n}}{3^{3n}}$

15. Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$.

16. Find the power series representation for the function $f(x) = \ln(3 - 2x)$ centered at 0.

17. Find the Taylor series for $f(x) = xe^x$ at $x = 2$.

18. Find the Maclaurin series for $f(x) = x \sin(x^3)$.

19. Find radius and center of sphere given by the equation $x^2 + y^2 + z^2 = 6x + 4y + 10z$

20. Find the angle between the vectors $\vec{a} = \vec{\imath} + \vec{\jmath} + 2\vec{k}$ and $\vec{b} = 2\vec{\jmath} - 3\vec{k}$.

21. Find the directional cosines for the vector $\vec{a} = -2\vec{\imath} + 3\vec{\jmath} + \vec{k}$.

22. Find the scalar and the vector projections of the vector $< 2, -3, 1 >$ onto the vector $< 1, 6, -2 >$.

23. Given vectors $\vec{a} =< -2, 3, 4 >$ and $\vec{b} =< 1, 0, 3 >$. Find $\vec{a} \times \vec{b}$.

24. Find the volume of the parallelepiped determined by vectors $\vec{a} =< 1, 0, 6 >$, $\vec{b} =< 2, 3, -8 >$, and $\vec{c} =< 8, -5, 6 >$.

25. Represent the point with Cartesian coordinates $(2\sqrt{3}, -2)$ in terms of polar coordinates.

26. Sketch the curve $r = \sin 5\theta$.