Section 10.6. **Representations of functions as a power series**

Geometric series.

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad |x| < 1.$$

Example 1. Find a power series representation for the function and determine the interval of convergence.

1. $\dfrac{1}{1+x}$

2. $\dfrac{1}{4-x^2}$

3. $\dfrac{2}{1+4x^2}$

4. $\dfrac{1+x^2}{1-x^2}$

Term-by-term differentiation and integration. If the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ has radius of convergence $R > 0$, then the function f defined by

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + ... + c_n(x-a)^n + ... = \sum_{n=0}^{\infty} c_n(x-a)^n$$

is differentiable (and therefore continuous) on the interval $(a-R, a+R)$ and

$$f'(x) = c_1 + 2c_2(x-a) + ... + nc_n(x-a)^{n-1} + ... = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$$

$$\int f(x)dx = C + c_0(x-a) + c_1\frac{(x-a)^2}{2} + ... + c_n\frac{(x-a)^{n+1}}{n+1} + ... =$$

$$C + \sum_{n=0}^{\infty} c_n\frac{(x-a)^{n+1}}{n+1}$$

The radii of convergence of these series are R. This does not mean that the interval of convergence remains the same.

Example 2. Find a power series representation for the function and determine the radius of convergence.

1. $\dfrac{1}{(1+x)^2}$

2. $\ln(4+x)$

Example 3. Evaluate an indefinite integral $\int \tan^{-1}(x^2)\ dx$ as a power series.

Example 4. Use a power series to approximate the integral

$$\int_0^{1/2} \tan^{-1}(x^2)\ dx$$

to six decimal places.