Section 6.5 **The Substitution Rule**

The substitution rule for indefinite integrals. If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Example 1. Evaluate each integral:

1. $\int \sin 5x dx$

2. $\displaystyle\int \frac{dx}{\sqrt{3x+1}}$

3. $\int x^2 e^{x^3} dx$

4. $\displaystyle\int \frac{2x^2+4x}{x^3+3x^2-4}dx$

5. $\displaystyle\int \frac{x + \arcsin x}{\sqrt{1-x^2}} dx$

The substitution rule for definite integrals. If $g'(x)$ is continuous on $[a, b]$ and f is continuous on the range of g, then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

Example 2. Evaluate the integral:

1. $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

2. $\displaystyle\int_0^1 \frac{xdx}{\sqrt{1+x^4}}$

3. $\displaystyle\int_{-3/2}^{0} \frac{x\ dx}{\sqrt{1-2x}}$

Integrals of symmetric functions. Suppose f is continuous on $[-a, a]$.

- If f is **even**, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$
- If f is **odd**, then $\int_{-a}^{a} f(x)dx = 0$

Example 3. Evaluate the integral $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6} dx$.