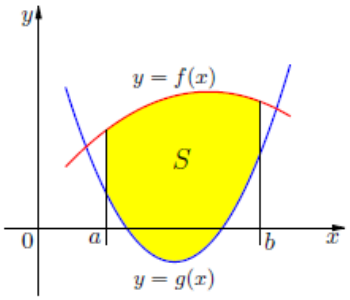
Chapter 7. **Applications of integration**
Section 7.1 **Areas between curves**

The area of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a, b]$, is

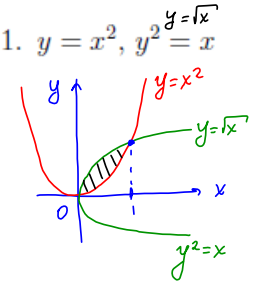
[top] - [bottom]

$$A = \int_a^b [f(x) - g(x)]dx$$



Example 1. Find the area of the region bounded by

1. $y = x^2$, $y^2 = x$



$$\text{Area} = \int_0^1 \Big([\text{top}] - [\text{bottom}]\Big)\,dx$$

$$= \int_0^1 \left(\sqrt{x} - x^2\right) dx$$

$$= \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)_0^1 = \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}}$$

Points of intersection:

$(x^2)^2 = (\sqrt{x})^2$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x_1 = 0, \quad x_2 = 1$

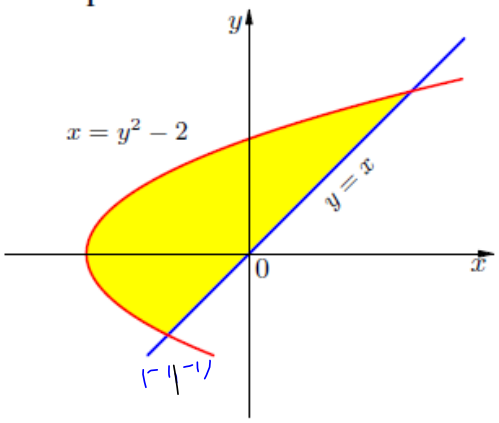
2. $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \pi/2$

3. $y = x^2 + 1$, $y = 3 - x^2$, $x = -1$, $x = 2$

In general case, the area between the curves $y = f(x)$, $y = g(x)$ and between $x = a$ and $x = b$, is

$$A = \int_a^b |f(x) - g(x)|dx$$

Example 2. Find the area of the shaded region.



If a region is bounded by curves with equations $x = f(y)$, $x = g(y)$, $y = c$ and $y = d$, where f and g are continuous functions and $f(y) \geq g(y)$ for all y in $[c, d]$, then its area is

$$A = \int_c^d [f(y) - g(y)]dy$$

Find the area of the region bounded by the parabola $y = x^2$, tangent line to this parabola at $(1, 1)$, and the x-axis.