- Quiz 3 over 7.3, 7.4, 7.5
- OFFICE HOURS TOMORROW 12:30 - 2:00 PM
- HW over 7.3, 7.4 due TOMORROW, FEB. 10, 11:55 PM

Chapter 8. Techniques of integration

Section 8.1 Integration by parts

The formula for integration by parts for indefinite integrals is

$$\boxed{\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx}$$

The formula for integration by parts for definite integrals is

$$\boxed{\int_a^b u(x)v'(x)dx = u(x)v(x)]_a^b - \int_a^b u'(x)v(x)dx}$$

Example 1. Find the integral.

$\boxed{\int \cos ax\,dx = \frac{1}{a}\sin ax + C}$

1. $\int x\cos 3x\,dx$ $\left| \begin{array}{ll} u = x & u' = 1 \\ v' = \cos 3x & v = \frac{1}{3}\sin 3x \end{array} \right|$ $= \overset{u}{x}\left(\overset{v}{\frac{1}{3}\sin 3x}\right) - \int \overset{u'}{(1)}\left(\overset{v}{\frac{1}{3}\sin 3x}\right)dx$

$\boxed{\int x^m \left\{\begin{array}{l} \cos ax \\ \sin bx \\ e^{cx} \end{array}\right\} dx \quad \Big| u = x^m}$

$\boxed{\int \sin bx\,dx = \frac{-1}{b}\cos bx + C}$

$= \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$

$= \frac{1}{3}x\sin 3x - \frac{1}{3}\left(-\frac{1}{3}\right)\cos 3x + C$

$= \boxed{\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C}$

2. $\int 1\cdot\ln x\,dx$ $= \left| \begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{array} \right|$ $= \overset{v}{x}\overset{u}{\ln x} - \int \overset{u'}{\frac{1}{x}}\,\overset{v}{x}\,dx$

$\boxed{\int x^m \left(\begin{array}{l} \ln ax \\ \arcsin bx \\ \arctan cx \end{array}\right) dx \quad \left| \begin{array}{l} x^m = v' \\ v = \frac{x^{m+1}}{m+1} \end{array} \right.}$

$= x\ln x - \int dx$

$= \boxed{x\ln x - x + C}$

3. $\int_0^1 (t^2 + 2t + 3)e^t dt$ $\left| \begin{array}{ll} u = t^2+2t+3 & u' = 2t+2 \\ v' = e^t & v = e^t \end{array} \right|$ $= (t^2+2t+3)e^t\Big]_0^1 - \int_0^1 (2t+2)e^t dt$

$= (1+2+3)e - 3e^0 - 2\int_0^1 (t+1)e^t dt$ $\left| \begin{array}{ll} u = t+1 & u' = 1 \\ v' = e^t & v = e^t \end{array} \right|$

$= 6e - 3 - 2\left\{(t+1)e^t\Big]_0^1 - \int_0^1 e^t dt\right\}$

$= 6e - 3 - 2\left\{2e - e^0 - e^t\Big]_0^1\right\} = 6e - 3 - 2\left(2e - \cancel{1} - e + \cancel{1}\right)$

$= 6e - 3 - 2e = \boxed{4e - 3}$

4. $\int 1\cdot\sin^{-1} x\, dx$ $\quad \left| \begin{array}{ll} u=\sin^{-1}x & u'=\frac{1}{\sqrt{1-x^2}} \\ v'=1 & v=x \end{array} \right| = x\sin^{-1}x - \int x \frac{1}{\sqrt{1-x^2}}\,dx \quad \left| \begin{array}{l} w=1-x^2 \\ dw=-2x\,dx \\ x\,dx=-\frac{dw}{2} \end{array} \right|$

$$= x\sin^{-1}x - \int \frac{-\frac{dw}{2}}{\sqrt{w}} = x\sin^{-1}x + \frac{1}{2}\int w^{-1/2}\,dw$$

$$= x\sin^{-1}x + \frac{1}{2}\,\frac{w^{1/2}}{1/2} + C$$

$$= x\sin^{-1}x + w^{1/2} + C = \boxed{x\sin^{-1}x + \sqrt{1-x^2} + C}$$

5. $\int e^x \cos x\, dx$ $\quad \left| \begin{array}{ll} u=\cos x & u'=-\sin x \\ v'=e^x & v=e^x \end{array} \right| = e^x\cos x - \int(-\sin x)\,e^x\,dx$

$$= e^x\cos x + \int \sin x\, e^x\,dx \quad \left| \begin{array}{ll} u=\sin x & u'=\cos x \\ v'=e^x & v=e^x \end{array} \right|$$

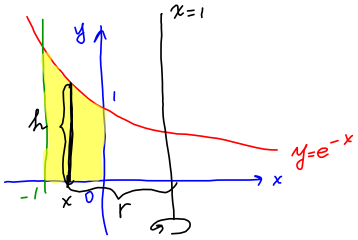
$$= e^x\cos x + e^x\sin x - \int e^x\cos x\,dx$$

$$\int e^x\cos x\,dx = e^x\cos x + e^x\sin x - \int e^x\cos x\,dx$$

$$\frac{2\int e^x\cos x\,dx}{2} = \frac{e^x\cos x + e^x\sin x}{2}$$

$$\boxed{\int e^x\cos x\,dx = \frac{1}{2}\left(e^x\cos x + e^x\sin x\right) + C}$$

Example 2. Use the methods of cylindrical shells to find the volume of a solid generated by rotating the region bounded by $y = e^{-x}$, $\underbrace{y = 0}_{x\text{-axis}}$, $x = -1$, $\underbrace{x = 0}_{y\text{-axis}}$ about $x = 1$.



Integrate for x. $\quad -1 \le x \le 0$

$$V = 2\pi \int_{-1}^{0} [\text{radius}][\text{height}]\,dx$$

$[\text{radius}] = 1 - x$

$[\text{height}] = e^{-x}$

$$V = 2\pi \int_{-1}^{0} (1-x)e^{-x}\,dx \quad \left|\begin{array}{ll} u = 1-x & u' = -1 \\ v' = e^{-x} & v = -e^{-x} \end{array}\right| = 2\pi\left\{(1-x)(-e^{-x})\Big]_{-1}^{0} - \int_{-1}^{0} (-1)(-e^{-x})\,dx\right\}$$

$$= 2\pi\left[-e^{0} - (2)(-e^{1}) - \int_{-1}^{0} e^{-x}\,dx\right]$$

$$= 2\pi\left(2e - 1 + e^{-x}\Big]_{-1}^{0}\right) = 2\pi\left(2e - 1 + e^{0} - e\right)$$

$$= 2\pi(2e - 1 + 1 - e)$$

$$= \boxed{2\pi e}$$