Chapter 8. **Techniques of integration**
Section 8.3 **Trigonometric substitution**

Assume that g is one-to-one function (g^{-1} exists). Then

$$\int f(x)dx = \int f(g(t))g'(t)dt$$

This kind of substitution is called *inverse substitution*.

Table of trigonometric substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin t, -\pi/2 \le t \le \pi/2$	$1 - \sin^2 t = \cos^2 t$
$\sqrt{a^2 + x^2}$	$x = a\tan t, -\pi/2 < t < \pi/2$	$1 + \tan^2 t = \sec^2 t$
$\sqrt{x^2 - a^2}$	$x = a\sec t, 0 \le t \le \pi/2$ or $\pi \le t \le 3\pi/2$	$\sec^2 t - 1 = \tan^2 t$

Example Find

(a) $\displaystyle\int x\sqrt{4 - x^2}dx$

(b) $\displaystyle\int \frac{x^3}{\sqrt{x^2 + 4}}dx$

(c) $\displaystyle\int \frac{dx}{x^2\sqrt{16x^2-9}}$

(d) $\displaystyle\int \frac{dx}{\sqrt{x^2+4x+8}}$

(e) $\int e^t\sqrt{9-e^{2t}}\ dt$