Review for Test 3, covering 10.3–10.7, 10.9, 11.1

8. Which of the following series converges absolutely?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\pi^3 n^2)}{n^2\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[4]{n}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{\ln(n+1)}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+4}{n^{11}+n^7+n+1}$

9. Suppose that the power series $\sum_{n=1}^{\infty} c_n(x-4)^n$ has the radius of convergence 4. Consider the following pair of series:

$$(I) \quad \sum_{n=1}^{\infty} c_n 5^n \qquad (II) \quad \sum_{n=1}^{\infty} c_n 3^n.$$

Which of the following statements is true?

(a) (I) is convergent, (II)is divergent

(b) Neither series is convergent

(c) Bith series are convergent

(d) (I) is divergent, (II) is convergent

(e) no conclusion can be drawn about either series.

10. Show that the series $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ converges. Then find un upper bound on the error in using s_{10} to approximate the series. (Note that $\ln 2 > 1/2$.)

11. If we represent $\dfrac{x^2}{4+9x^2}$ as a power series centered at $a = 0$, what is the associated radius of convergence?

12. Find the radius and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n(3x-1)^n}{n}$.

13. Which of the following statements is TRUE?

(a) If $a_n > 0$ for $n \geq 1$ and $\displaystyle\sum_{n=1}^{\infty}(-1)^n a_n$ converges then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(b) If $a_n > 0$ for $n \geq 1$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges then $\displaystyle\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(c) If $\displaystyle\lim_{n\to\infty} a_n = 0$ then $\displaystyle\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(d) If $a_n > 0$ for $n \geq 1$ and $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{e}{2}$ then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

14. Find a Maclaurin series representation for $\dfrac{e^x - 1 - x}{x^2}$.

15. (a) Find a Maclaurin series representation for $f(x) = \sin\left(\dfrac{x^2}{4}\right)$

(b) Write $\displaystyle\int_0^1 \sin\left(\frac{x^2}{4}\right) \mathrm{d}x$ as an infinite series.

16. Let $f(x) = e^{5-x}$. Give the fourth degree Taylor polynomial for $f(x)$ centered around $a = 5$.

17. Find a Maclaurin series of $f(x) = \ln(2 - x)$ and the associated radius of convergence.

18. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 3^n}$ converges to s. Use the Alternating Series Theorem to estimate $|s - s_6|$.

19. Determine the radius and the center of the sphere given by the equation

$$x^2 + y^2 + z^2 + 2y + z - 1 = 0.$$