

1. Find the volume generated by rotating the region by the given curves.

(a) $y = \frac{1}{x^4}$, $x = \frac{1}{2}$, $x = 1$, $y = 0$. Rotate about the y -axis.

(b) $y = x^3$, $x = 0$, $y = 27$. Rotate about the x -axis.

(c) $y = \sqrt{x}$, $y = 4x$. Rotate about the line $x = -1$.

(d) $y = x^2$ and $x = y^2$. Rotate about the line $y = 3$.

2. A force of $F(x) = x^4 - \sin(4\pi x) + 12$ (where x is in meters) acts on an object. What is the work required to move the object from $x = 3$ to $x = 5$?
3. If the force required to stretch a spring 3 ft beyond its natural length is 12 lb, how much work is needed to stretch it beyond its natural length?
4. A spring has a natural length of 20 cm. If a 10 J work is required to keep it stretched to a length 25 cm, how much work is done in stretching the spring from 30 cm to 80 cm?

5. A heavy rope 40 ft long, weighs 0.4 lb/ft and hangs over the edge of a tall building. How much work is done in pulling the rope to the top of the building?

6. A uniform cable hanging over the edge of a tall building is 6 m long and weighs 20 kg. If 25 kg weight is attached to the cable, how much work is required to pull 2 m of the cable to the top of the building?

7. A tank of water is 20 ft long and has a vertical cross section in a shape of an equilateral triangle with sides 2 ft long. The tank is filled with water to a depth of 18 inches. Determine the amount of work needed to pump all of the water to the top of the tank. The weight of water is 62.5 lb/ft^3 .

8. A spherical tank with radius 8 m is half full of water. The water pumped out of the spout of the top of the tank that is 75 cm high. Find the work done needed to pump out the water through the spout until the water level is 4 m from the bottom. The density of water is 1000 kg/m^3 .

9. Determine the average value of the function $f(x) = \frac{\sqrt{3}-1}{1+x^2}$ over the interval $[1, \sqrt{3}]$.

10. Find the value(s) a such that the average value of the function $f(x) = 3x^2 - 2x - 3$ over the interval $[a, 0]$ is equal to 2.