Math 152/172 **WEEK in REVIEW 5** Spring 2016

Sections 8.3, 8.4, 8.9

Section 8.3

Table of trigonometric substitutions

Expression	Substitution	Identity
$\sqrt{a^2-x^2}$	$x=a\sin t, -\pi/2\leq t\leq \pi/2$	$1-\sin^2 t=\cos^2 t$
$\sqrt{a^2+x^2}$	$x=a\tan t, -\pi/2<t<\pi/2$	$1+\tan^2 t=\sec^2 t$
$\sqrt{x^2-a^2}$	$x=a\sec t, 0\leq t\leq \pi/2$ or $\pi\leq t\leq 3\pi/2$	$\sec^2 t-1=\tan^2 t$

1. Evaluate the integral

(a) $\int \frac{x^2}{\sqrt{4+x^2}}\,dx$

$\sqrt{x^2+2^2}$

$x=2\tan t \Rightarrow \tan t=\frac{x}{2}$

$dx=2\sec^2 t\,dt$

$\sqrt{4+x^2}=\sqrt{4+4\tan^2 t}$

$=\sqrt{4(1+\tan^2 t)}$

$=\sqrt{4\sec^2 t}$

$\sqrt{4+x^2}=2\sec t$

$\sec t=\frac{\sqrt{4+x^2}}{2}$

$=\int \frac{4\tan^2 t}{\cancel{2}\cancel{\sec t}}\,\cancel{2}\sec^{\cancel{2}} t\,dt$

$=4\int \tan^2 t\sec t\,dt$

$=4\int (\sec^2 t-1)\sec t\,dt$

$=4\int (\sec^3 t-\sec t)\,dt$

$\boxed{\begin{array}{l}\int \sec^3 x\,dx=\frac{1}{2}\left(\sec x\tan x+\ln|\sec x+\tan x|\right)+C\\ \int \sec x\,dx=\ln|\sec x+\tan x|+C\end{array}}$

$=4\left(\frac{1}{2}\left(\sec t\tan t+\ln|\sec t+\tan t|\right)-\ln|\sec t+\tan t|\right)+C$

$=\frac{4}{2}\left(\sec t\tan t-\ln|\sec t+\tan t|\right)+C$

$=2\left(\frac{\sqrt{4+x^2}}{2}\frac{x}{2}-\ln\left|\frac{\sqrt{4+x^2}}{2}+\frac{x}{2}\right|\right)+C$

$=\boxed{\frac{x\sqrt{4+x^2}}{2}-2\ln\left|\frac{\sqrt{4+x^2}+x}{2}\right|+C}$

(b) $\int \frac{x}{\sqrt{6x - x^2}}\,dx$

Complete the square

$6x - x^2 = -(x^2 - 6x) = -((x^2 - 6x + 9) - 9) = -(x^2 - 6x + 9) - (-9)$

$= -(x^2 - 6x + 9) + 9 = 9 - (x-3)^2$

$= \int \frac{x}{\sqrt{9-(x-3)^2}}\,dx$

$x - 3 = 3\sin t$ | $\sin t = \frac{x-3}{3}$

$x = 3 + 3\sin t$ | $t = \arcsin\frac{x-3}{3}$

$dx = 3\cos t\,dt$

$\sqrt{9-(x-3)^2} = \sqrt{9 - 9\sin^2 t}$

$= \sqrt{9(1-\sin^2 t)}$

$= \sqrt{9\cos^2 t}$

$\sqrt{9-(x-3)^2} = 3\cos t$

$= \int \frac{3 + 3\sin t}{\cancel{3\cos t}}\,\cancel{3\cos t}\,dt$

$= \int (3 + 3\sin t)\,dt$

$= 3t - 3\cos t + C$

$= \boxed{3\arcsin\frac{x-3}{3} - \sqrt{9-(x-3)^2} + C}$

(c) $\displaystyle\int \frac{dx}{x^2\sqrt{25x^2-9}} = \int \frac{dx}{x^2\sqrt{(5x)^2-3^2}}$

$5x = 3\sec t$

$x = \frac{3}{5}\sec t \Rightarrow \cos t = \frac{3}{5x}$

$dx = \frac{3}{5}\sec t \tan t \, dt$

$\sqrt{25x^2-9} = \sqrt{9\sec^2 t - 9} = \sqrt{9(\sec^2 t - 1)}$

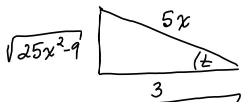
$= 3\tan t$

$$= \int \frac{\cancel{\frac{3}{5}\sec t}\ \cancel{\tan t}\ dt}{(\frac{3}{5}\sec t)^{\cancel{2}}\ \cancel{3\tan t}}$$

$$= \int \frac{5\,dt}{9\sec t} = \frac{5}{9}\int \cos t \, dt = \frac{5}{9}\sin t + C$$

$$= \frac{5}{9}\cdot\frac{\sqrt{25x^2-9}}{5x} + C$$

$$= \boxed{\frac{\sqrt{25x^2-9}}{9x} + C}$$



$\cos t = \frac{3}{5x}$

$\sin t = \frac{\sqrt{25x^2-9}}{5x}$

Section 8.4

To integrate any rational function $f(x) = \dfrac{P(x)}{Q(x)}$, where $P(x) = a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n$, $Q(x) = b_0x^m + b_1x^{m-1} + ... + b_m$:

STEP 1. If f is improper ($n \geq m$), then we must divide P into Q by long divisions until a remainder $R(x)$ is obtained. The division statement is

$$f(x) = \frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$$

STEP 2. Factor the denominator $Q(x)$ as far as possible. It can be shown that any polynomial Q can be factored as a product of *linear factors* of the form $ax + b$ and *irreducible quadratic factors* (of the form $ax^2 + bx + c$, where $b^2 - 4ac < 0$).

STEP 3. Express the proper rational function $\dfrac{R(x)}{Q(x)}$ as a sum of **partial fractions** of the form

$$\frac{A}{(ax+b)^i} \quad \text{or} \quad \frac{Ax+B}{(ax^2+bx+c)^j}$$

Factor	Corresponding fraction
$ax+b$	$\dfrac{A}{ax+b}$
$(ax+b)^r, r>1$	$\dfrac{A_1}{ax+b} + \dfrac{A_2}{(ax+b)^2} + \cdots + \dfrac{A_r}{(ax+b)^r}$
$ax^2+bx+c, b^2-4ac<0$	$\dfrac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^r, b^2-4ac<0, r>1$	$\dfrac{A_1x+B_1}{ax^2+bx+c} + \dfrac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \dfrac{A_rx+B_r}{(ax^2+bx+c)^r}$

2. Evaluate the integral

(a) $\displaystyle\int \frac{7}{(x-2)(x+5)}\,dx$

$$\frac{7}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5}$$

$$\frac{7}{\cancel{(x-2)(x+5)}} = \frac{A(x+5)+B(x-2)}{\cancel{(x-2)(x+5)}}$$

$$7 = A(x+5) + B(x-2)$$

$x=-5$: $7 = -7B \Rightarrow \boxed{B=-1}$

$x=2$: $7 = 7A \Rightarrow \boxed{A=1}$

$$\int \frac{7\,dx}{(x-2)(x+5)} = \int \left[\frac{1}{x-2} - \frac{1}{x+5}\right] dx$$

$$= \ln|x-2| - \ln|x+5| + C$$

$$= \boxed{\ln\left|\frac{x-2}{x+5}\right| + C}$$

(b) $\int \frac{x^5}{(x-2)^2}\,dx$

$(x-2)^2 = x^2 - 4x + 4$

$$\begin{array}{r|l} & x^3 + 4x^2 + 12x + 32 \\ \hline x^2-4x+4 & x^5 \\ & -\ x^5 - 4x^4 + 4x^3 \\ \hline & 4x^4 - 4x^3 \\ & -\ 4x^4 - 16x^3 + 16x^2 \\ \hline & 12x^3 - 16x^2 \\ & -\ 12x^3 - 48x^2 + 48x \\ \hline & 32x^2 - 48x \\ & -\ 32x^2 - 128x + 128 \\ \hline & 80x - 128 \end{array}$$

$$\int \frac{x^5}{x^2-4x+4}dx = \int\left[x^3+4x^2+12x+32+\frac{80x-128}{x^2-4x+4}\right]dx$$

$$= \frac{x^4}{4} + \frac{4x^3}{3} + \frac{12x^2}{2} + 32x + \int \frac{80x-128}{x^2-4x+4}dx$$

$$\int \frac{80x-128}{(x-2)^2}dx = 16\int \frac{5x-8}{(x-2)^2}dx$$

Partial fractions:

$$\frac{5x-8}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$$\frac{5x-8}{\cancel{(x-2)^2}} = \frac{A(x-2)+B}{\cancel{(x-2)^2}}$$

$5x-8 = A(x-2)+B$

$x=2$: $10-8 = B \Rightarrow \boxed{B=2}$

$x=0$: $-8 = -2A + B$

$2A = B+8$

$2A = 10 \Rightarrow \boxed{A=5}$

$$16\int \frac{5x-8}{(x-2)^2}dx = 16\int\left[\frac{5}{x-2} + \frac{2}{(x-2)^2}\right]dx$$

$$= 16\left(5\ln|x-2| - \frac{2}{x-2}\right) + C$$

$$\boxed{\int \frac{x^5}{(x-2)^2}dx = \frac{x^4}{4} + \frac{4x^3}{3} + 6x^2 + 32x + 80\ln|x-2| - \frac{32}{x-2} + C}$$

(c) $\int \frac{x^2-3x+7}{(x-1)(x^2+1)}dx$

$$\frac{x^2-3x+7}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2-3x+7}{\cancel{(x-1)(x^2+1)}} = \frac{A(x^2+1)+(Bx+C)(x-1)}{\cancel{(x-1)(x^2+1)}}$$

$$x^2-3x+7 = A(x^2+1)+(Bx+C)(x-1)$$

$x=1$: $\quad 1-3+7 = 2A \Rightarrow \boxed{A=\frac{5}{2}}$

$x=0$: $\quad 7 = A-C \Rightarrow C=-7+A=-7+\frac{5}{2} = \boxed{-\frac{9}{2}=C}$

$x=-1$: $\quad 1+3+7 = 2A+(-B+C)(-2)$

$11 = 2A+2B-2C \Rightarrow 2B = 11-2A+2C$
$= 11-5-9 = -3 \Rightarrow \boxed{B=-\frac{3}{2}}$

$x^2-3x+7 = A(x^2+1)+(Bx+C)(x-1)$
$x^2-3x+7 = Ax^2+A+Bx^2-Bx+Cx-C$
$x^2-3x+7 = x^2(A+B)+x(C-B)+(A-C)$

Equate coefficients to the corresponding powers of x:

$$\begin{cases} x^2: & 1 = A+B \\ x: & -3 = C-B \\ 1: & 7 = A-C \end{cases}$$

$$\int \frac{x^2-3x-7}{(x-1)(x^2+1)}dx = \int\left[\frac{\frac{5}{2}}{x-1} + \frac{-\frac{3}{2}x-\frac{9}{2}}{x^2+1}\right]dx$$

$$= \frac{5}{2}\ln|x-1| + \int\left(-\frac{3}{2}\right)\frac{x}{x^2+1}dx - \frac{9}{2}\int\frac{1}{x^2+1}dx$$

$u = x^2+1$
$du = 2x\,dx$
$x\,dx = \frac{du}{2}$

$$= \frac{5}{2}\ln|x-1| - \frac{3}{4}\int\frac{du}{u} - \frac{9}{2}\arctan x + C$$

$$= \frac{5}{2}\ln|x-1| - \frac{3}{4}\ln|u| - \frac{9}{2}\arctan x + C$$

$$= \boxed{\frac{5}{2}\ln|x-1| - \frac{3}{4}\ln|x^2+1| - \frac{9}{2}\arctan x + C}$$

(d) $\int \frac{dx}{(x^2+1)(x^2+x+1)}$

$$\frac{1}{(x^2+1)(x^2+x+1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+x+1}$$

$$\frac{1}{\cancel{(x^2+1)(x^2+x+1)}} = \frac{(Ax+B)(x^2+x+1)+(Cx+D)(x^2+1)}{\cancel{(x^2+1)(x^2+x+1)}}$$

$$1 = Ax^3+Ax^2+Ax+Bx^2+Bx+B+Cx^3+Cx+Dx^2+D$$

$$1 = x^3(A+C)+x^2(A+B+D)+x(A+B+C)+(B+D)$$

Equate coefficients:

$$\begin{cases} x^3: & 0 = A+C \\ x^2: & 0 = A+B+D \\ x: & 0 = A+B+C \\ 1: & 1 = B+D \end{cases}$$

$B+D=1$

$A+B+D=0 \Rightarrow A = -(B+D) = \boxed{-1 = A}$

$A+C=0 \Rightarrow C=-A = \boxed{1=C}$

$A+B+C=0 \Rightarrow B = -(A+C) = \boxed{0=B}$

$D = 1-B = \boxed{1=D}$

$$\int \frac{dx}{(x^2+1)(x^2+x+1)} = \int\left[\frac{-x}{x^2+1} + \frac{x+1}{x^2+x+1}\right]dx$$

$$\int \frac{-x}{x^2+1}dx \quad \left| \begin{array}{l} u=x^2+1 \\ du=2x\,dx \end{array}\right| = -\frac{1}{2}\int\frac{du}{u} = -\frac{1}{2}\ln|u|+C = -\frac{1}{2}\ln|x^2+1|+C$$

x^2+x+1

$= x^2+2\cdot\frac{1}{2}x+\frac{1}{4}-\frac{1}{4}+1$

$= (x+\frac{1}{2})^2+\frac{3}{4}$

$$\boxed{\int\frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}+C}$$

$$\int\frac{x+1}{x^2+x+1}dx = \int\frac{x+1}{(x+\frac{1}{2})^2+\frac{3}{4}}dx = \int\frac{(x+\frac{1}{2})+\frac{1}{2}}{(x+\frac{1}{2})^2+\frac{3}{4}}dx \quad \left|\begin{array}{l} u = x+\frac{1}{2} \\ du = dx\end{array}\right|$$

$$= \int\frac{u+1/2}{u^2+3/4}du = \int\frac{u}{u^2+3/4}du + \frac{1}{2}\int\frac{du}{u^2+\frac{3}{4}}$$

$$\left|\begin{array}{l} v=u^2+3/4 \\ dv=2u\,du\end{array}\right|$$

$$= \frac{1}{2}\int\frac{dv}{v} + \frac{1}{2}\cdot\frac{1}{\frac{\sqrt{3}}{2}}\arctan\frac{x}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2}\ln|v| + \frac{1}{\sqrt{3}}\arctan\frac{2x}{\sqrt{3}}+C$$

$$= \frac{1}{2}\ln\left|u^2+\frac{3}{4}\right| + \frac{1}{\sqrt{3}}\arctan\frac{2x}{\sqrt{3}}+C$$

$$= \frac{1}{2}\ln\left|(x+\tfrac{1}{2})^2+\frac{3}{4}\right| + \frac{1}{\sqrt{3}}\arctan\frac{2x}{\sqrt{3}}+C$$

$$\boxed{\int\frac{dx}{(x^2+1)(x^2+x+1)} = -\frac{1}{2}\ln|x^2+1| + \frac{1}{2}\ln\left|(x+\tfrac{1}{2})^2+\frac{3}{4}\right| + \frac{1}{\sqrt{3}}\arctan\frac{2x}{\sqrt{3}}+C}$$

3. Decompose into partial fractions the rational function without computing coefficient of the decomposition:

$$\frac{x-1}{(x+2)^3\underbrace{(x^2-2x+5)^2}_{\text{irreducible}}}$$

$$\boxed{\frac{x-1}{(x+2)^3(x^2-2x+5)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} + \frac{Dx+E}{x^2-2x+5} + \frac{Fx+G}{(x^2-2x+5)^2}}$$

Improper integral of Type 1: $\int_a^\infty f(x)dx = \lim_{t\to\infty}\int_a^t f(x)dx$ (likewise with $-\infty$), f is continuous

Improper integral of Type 2: $f(x)$ is discontinuous @ a, $\int_a^b f(x)dx = \lim_{t\to a^+}\int_t^b f(x)dx$ (likewise with b^-)

4. Compute the following integrals or show that they are diverge.

(a) $$\int_e^\infty \frac{dx}{x\ \ln^5 x} = \lim_{t\to\infty}\int_e^t \frac{dx}{x\ln^5 x} = \left| \begin{array}{l} u=\ln x \\ du=\frac{dx}{x} \\ e \to \ln e = 1 \\ t \to \ln t \end{array}\right| = \lim_{t\to\infty}\int_1^{\ln t}\frac{du}{u^5} = \lim_{t\to\infty}\left.\frac{u^{-4}}{-4}\right]_1^{\ln t}$$

$$= \lim_{t\to\infty}\left[-\frac{1}{4\cancel{(\ln t)^4}}\overset{0}{} + \frac{1}{4}\right] = \boxed{\frac{1}{4}} \quad \text{convergent}$$

$$\boxed{\int_a^b uv'dx = uv\Big]_a^b - \int_a^b vu'dx}$$

(b) $$\int_{-\infty}^0 (1+x)e^x\,dx = \lim_{t\to-\infty}\int_t^0 (1+x)e^x dx = \left| \begin{array}{ll} u=1+x & u'=1 \\ v'=e^x & v=e^x \end{array}\right|$$

$$= \lim_{t\to-\infty}\left((1+x)e^x\Big]_t^0 - \int_t^0 e^x dx\right) = \lim_{t\to-\infty}\left(1-(1+t)e^t - e^x\Big]_t^0\right)$$

$$= \lim_{t\to-\infty}\left(\underbrace{1-(1+t)e^t-1+e^t}_{1-e^t-te^t-1+e^t}\right) = -\lim_{t\to-\infty} te^t \quad |0\cdot\infty|$$

$$= -\lim_{t\to-\infty}\frac{t}{e^{-t}} \overset{\text{L'Hospital's}}{=} -\lim_{t\to-\infty}\frac{1}{-e^{-t}} = \boxed{0} \quad \text{convergent}$$

$$= +\lim_{t\to-\infty} e^t$$

$$\boxed{\int_a^\infty \frac{dx}{x^p} = \begin{cases} \text{converges, if } p>1 \\ \text{diverges, if } p\le 1 \end{cases}}$$

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{5x^4}{(x^5+3)^3}dx = \int_{-\infty}^{0} \frac{5x^4}{(x^5+3)^3}dx + \int_0^\infty \frac{5x^4}{(x^5+3)^3}dx$

$$\boxed{\frac{x^4}{x^{15}} = \frac{1}{x^{11}},\ p=11>1 \qquad \text{convergent}}$$

$$= \lim_{t\to-\infty}\int_t^0 \frac{5x^4}{(x^5+3)^3}dx + \lim_{s\to\infty}\int_0^s \frac{5x^4}{(x^5+3)^3}dx$$

$$\int \frac{5x^4}{(x^5+3)^3}dx \quad \left| \begin{array}{l} u=x^5+3 \\ du=5x^4dx \end{array} \right.$$

$$= \int \frac{du}{u^3} = \frac{u^{-2}}{-2}+C = -\frac{1}{2u^2}+C = -\frac{1}{2(x^5+3)^2}+C$$

$$= \lim_{t\to-\infty}\left(-\frac{1}{2(x^5+3)^2}\Big]_t^0\right) + \lim_{s\to\infty}\left(-\frac{1}{2(x^5+3)^2}\Big]_0^s\right)$$

$$= -\frac{1}{18} + \lim_{t\to-\infty}\cancel{\frac{1}{2(t^5+3)^2}}^{\,0} - \lim_{s\to\infty}\cancel{\frac{1}{2(s^5+3)^2}}^{\,0} + \frac{1}{18} = \boxed{0}$$

$$\int_0^1 \frac{dx}{x^p} = \begin{cases} \text{converges, if } p<1 \\ \text{diverges, if } p\ge 1 \end{cases}$$

(d) $\displaystyle\int_0^9 \frac{dx}{\sqrt[3]{x-4}}$ $\quad (p=\tfrac{1}{3})$ convergent.

$\dfrac{1}{\sqrt[3]{x-4}}$ has discontinuity @ $x=4$

$$= \int_0^4 \frac{dx}{\sqrt[3]{x-4}} + \int_4^9 \frac{dx}{\sqrt[3]{x-4}} = \lim_{s\to4^-}\int_0^s \frac{dx}{\sqrt[3]{x-4}} + \lim_{t\to4^+}\int_t^9 \frac{dx}{\sqrt[3]{x-4}}$$

$$= \lim_{s\to4^-} \frac{(x-4)^{-1/3+1}}{-1/3+1}\Big]_0^s + \lim_{t\to4^+} \frac{(x-4)^{-1/3+1}}{-1/3+1}\Big]_t^9$$

$$= \lim_{s\to4^-} \frac{3}{2}(x-4)^{2/3}\Big]_0^s + \lim_{t\to4^+} \frac{3}{2}(x-4)^{2/3}\Big]_t^9 = \lim_{s\to4^-}\left[\frac{3}{2}\cancel{(s-4)^{2/3}}^{\,0} - \frac{3}{2}(-4)^{2/3}\right]$$

$$+ \lim_{t\to4^+}\frac{3}{2}\left[(9-4)^{2/3} - \cancel{(t-4)^{2/3}}^{\,0}\right] = \boxed{-\frac{3}{2}\left(\sqrt[3]{16}\right) + \frac{3}{2}\left(\sqrt[3]{25}\right)}$$

5. Determine whether the given integrals converge or diverge using the Comparison Theorem.

(a) $\displaystyle\int_0^\infty \frac{dx}{x^7+e^{7x}}$

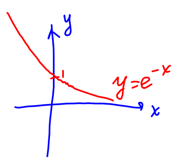
<u>Comparison Theorem.</u> If $f(x)$ and $g(x)$ are continuous on $[a,\infty)$ functions such that $f(x) \geq g(x)$ on $[a,\infty)$, then

(a) if $\int_a^\infty f(x)\,dx$ converges, then $\int_a^\infty g(x)\,dx$ is convergent.

(b) if $\int_a^\infty g(x)\,dx$ diverges, then $\int_a^\infty f(x)$ is divergent.

5. Determine whether the given integrals converge or diverge using the Comparison Theorem.

(a) $\displaystyle\int_0^\infty \frac{dx}{x^7 + e^{7x}}$



$$\frac{1}{x^7+e^{7x}} \leq \frac{1}{e^{7x}}$$

$$\int_0^\infty \frac{1}{e^{7x}}\,dx = \int_0^\infty e^{-7x}\,dx = \lim_{t\to\infty} \int_0^t e^{-7x}\,dx = -\frac{1}{7} \lim_{t\to\infty} e^{-7x}\Big]_0^t$$

$$= -\frac{1}{7}\left[\cancelto{0}{\lim_{t\to\infty} e^{-7t}} - \lim_{t\to\infty} e^0\right] = \boxed{\frac{1}{7}} \text{ convergent.}$$

$\int_0^\infty \frac{1}{e^{7x}}\,dx$ converges, by Comparison Thm part (a) $\int_0^\infty \frac{dx}{x^7+e^{7x}}$ converges

$$\int_1^\infty \frac{dx}{x^p} = \begin{cases} p>1, & \text{converges} \\ p\le 1, & \text{diverges.} \end{cases}$$

(b) $\displaystyle\int_5^\infty \frac{x^2}{x^{5/2}-x}dx$ $\boxed{\text{divergent}}$

$$\frac{x^2}{x^{5/2}} = \frac{1}{x^{1/2}} \quad p = 1/2 < 1$$

$$\frac{x^2}{x^{5/2}-x} \geq \frac{x^2}{x^{5/2}} = \frac{1}{x^{1/2}}$$

$\displaystyle\int_5^\infty \frac{dx}{x^{1/2}}$ diverges, so by Comparison Theorem part (b)

$\displaystyle\int_5^\infty \frac{x^2}{x^{5/2}-x}dx$ is divergent.

(c) $\displaystyle\int_{10}^\infty \frac{\sin^4(7x)}{x^7}dx$

$$\frac{0}{x^7} \le \underbrace{\frac{\sin^4(7x)}{x^7}}_{} \le \frac{1}{x^7}$$

$\displaystyle\int_{10}^\infty \frac{dx}{x^7}$ $(p=7>1)$ is convergent

By Comparison Thm. part (a), $\displaystyle\int_{10}^\infty \frac{\sin^4(7x)}{x^7}dx$ is convergent.