Sections 9.3, 9.4, 10.1

1. Find the length of the curve $x = \cos^3 t, y = \sin^3 t$, $0 \leq t \leq \pi/2$.

2. Find the length of the curve $x = \frac{1}{4}\ln y - \frac{1}{2}y^2$ from $y = 1$ to $y = e$.

3. A wire hanging between two poles (at $x = -10$ and $x = 10$) takes the shape of a catenary with equation

$$y = 2(e^{x/4} + e^{-x/4}).$$

   Find the length of the wire.

4. The curve $y = x^2$, $0 \leq x \leq 1$, is rotated about the y-axis. Find the area of the resulting surface.

5. The curve $x = 1 - \cos(2t)$, $y = 2t + \sin(2t)$, $0 \leq t \leq \pi/4$ is rotated about the x-axis. Find the area of the resulting surface.

6. Set up (but don't evaluate) the integral that gives the surface area obtaine by rotating the curve

$$x = \sin(\pi y^2/8), \quad 1 \leq y \leq 2,$$

   (a) about the x-axis
   (b) about the y-axis

7. The curve $x = \sin(at), y = \cos(at), 0 \leq t \leq \frac{\pi}{2a}$ is rotated about the x-axis (here a is an arbitrary positive constant). Find the area of the resulting surface.

8. Define the n-th term of the sequence $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots\right\}$ and find its limit.

9. Determine if the given sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5}$
   (b) $b_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5 + 11n^6}$
   (c) $c_n = \dfrac{12n^7 + 2012}{2012 - 12n^4 - 4n^5 - 9n^6}$

10. Determine if the sequence with the given general term ($n \geq 1$) converges or diverges. If it converges, find the limit.

   (a) $a_n = \ln(n^2 + 3) - \ln(7n^2 - 5)$
   (b) $z_n = \dfrac{1}{n^4}\sin\left(\dfrac{1}{n^5}\right)$
   (c) $y_n = \dfrac{(-1)^n}{n^3}$
   (d) $x_n = \dfrac{(-1)^n n}{3n + 33}$

11. Assuming that the sequence defined recursively by $a_n = 1$, $a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{9}{a_n}\right)$ is convergent, find its limit.

12. Determine whether the given sequence is increasing or decreasing.

   (a) $\{\arctan(n)\}_{n=1}^{\infty}$
   (b) $\{n - 2^n\}_{n=1}^{\infty}$