Sections 9.3, 9.4, 10.1

- If a curve C is defined by the equations $x = x(t), y = y(t), \quad a \le t \le b$, then $L = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$
- If a curve C is given by the equation $y = y(x), \quad a \le x \le b$, then $L = \int_a^b \sqrt{1 + [y'(x)]^2} dx$
- If a curve C is given by the equation $x = x(y), \quad c \le y \le d$, then $L = \int_c^d \sqrt{1 + [x'(y)]^2} dy$

1. Find the length of the curve $x = \cos^3 t, y = \sin^3 t$, $0 \le t \le \pi/2$.

2. Find the length of the curve $x = \frac{1}{4} \ln y - \frac{1}{2} y^2$ from $y = 1$ to $y = e$.

3. A wire hanging between two poles (at $x = -10$ and $x = 10$) takes the shape of a catenary with equation

$$y = 2(e^{x/4} + e^{-x/4}).$$

Find the length of the wire.

For rotation about the x-axis, the surface area formulas are:

- if a curve is given as $y = y(x)$, $a \le x \le b$, then $S.A. = 2\pi \int_a^b y(x)\sqrt{1 + [y'(x)]^2} dx$
- if a curve is described as $x = x(y)$, $c \le y \le d$, then $S.A. = 2\pi \int_c^d y\sqrt{1 + [x'(y)]^2} dy$
- if a curve is defined by $x = x(t)$,$y = y(t)$, $a \le t \le b$, then $S.A. = \int_a^b y(t)\sqrt{[x'(t)]^2 + [y'(t)]^2} dt$

For rotation about the y-axis, the surface area formulas are:

- if a curve is given as $y = y(x)$, $a \le x \le b$, then $S.A. = 2\pi \int_a^b x\sqrt{1 + [y'(x)]^2} dx$
- if a curve is described as $x = x(y)$, $c \le y \le d$, then $S.A. = 2\pi \int_c^d x(y0\sqrt{1 + [x'(y)]^2} dy$
- if a curve is defined by $x = x(t)$,$y = y(t)$, $a \le t \le b$, then $S.A. = \int_a^b x(t)\sqrt{[x'(t)]^2 + [y'(t)]^2} dt$

4. The curve $y = x^2$, $0 \le x \le 1$, is rotated about the y-axis. Find the area of the resulting surface.

5. The curve $x = 1 - \cos(2t)$, $y = 2t + \sin(2t)$, $0 \le t \le \pi/4$ is rotated about the x-axis. Find the area of the resulting surface.

6. Set up (but don't evaluate) the integral that gives the surface area obtaine by rotating the curve

$$x = \sin(\pi y^2/8), \quad 1 \leq y \leq 2,$$

(a) about the x-axis

(b) about the y-axis

7. The curve $x = \sin(at), y = \cos(at), 0 \leq t \leq \frac{\pi}{2a}$ is rotated about the x-axis (here a is an arbitrary positive constant). Find the area of the resulting surface.

8. Define the n-th term of the sequence $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots\right\}$ and find its limit.

9. Determine if the given sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5}$

   (b) $b_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5 + 11n^6}$

   (c) $c_n = \dfrac{12n^7 + 2012}{2012 - 12n^4 - 4n^5 - 9n^6}$

10. Determine if the sequence with the given general term ($n \geq 1$) converges or diverges. If it converges, find the limit.

   (a) $a_n = \ln(n^2 + 3) - \ln(7n^2 - 5)$

(b) $z_n = \frac{1}{n^4} \sin\left(\frac{1}{n^5}\right)$

(c) $y_n = \frac{(-1)^n}{n^3}$

(d) $x_n = \frac{(-1)^n n}{3n + 33}$

11. Assuming that the sequence defined recursively by $a_n = 1$, $a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right)$ is convergent, find its limit.

12. Determine whether the given sequence is increasing or decreasing.

(a) $\{\arctan(n)\}_{n=1}^{\infty}$

(b) $\{n - 2^n\}_{n=1}^{\infty}$